University of the Peloponnese
Electrical and Computer
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DIGITAL SIGNAL PROCESSING

Solved Examples

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SET #12 - Structures of discrete-time systems

- IIR filters
- FIR filters
- Lattice Filters

1. IIR filters

Example 1

An LSI system is described by the transfer function:

$$H(z) = \frac{1 + 0.9z^{-1}}{(1 + 0.1z^{-1} + 0.5z^{-2})(1 - 0.6z^{-1})}$$

(a) Draw the step diagrams of straight form I and II.

(b) For each format calculate the number of multiplications and additions required to calculate each output sample, as well as the number of delay registers.

Answer: (a) We do the operations on the denominator, so the transfer function is written:

$$H(z) = \frac{1 + 0.9z^{-1}}{1 + 0.7z^{-1} + 0.44z^{-2} - 0.3z^{-3}}$$

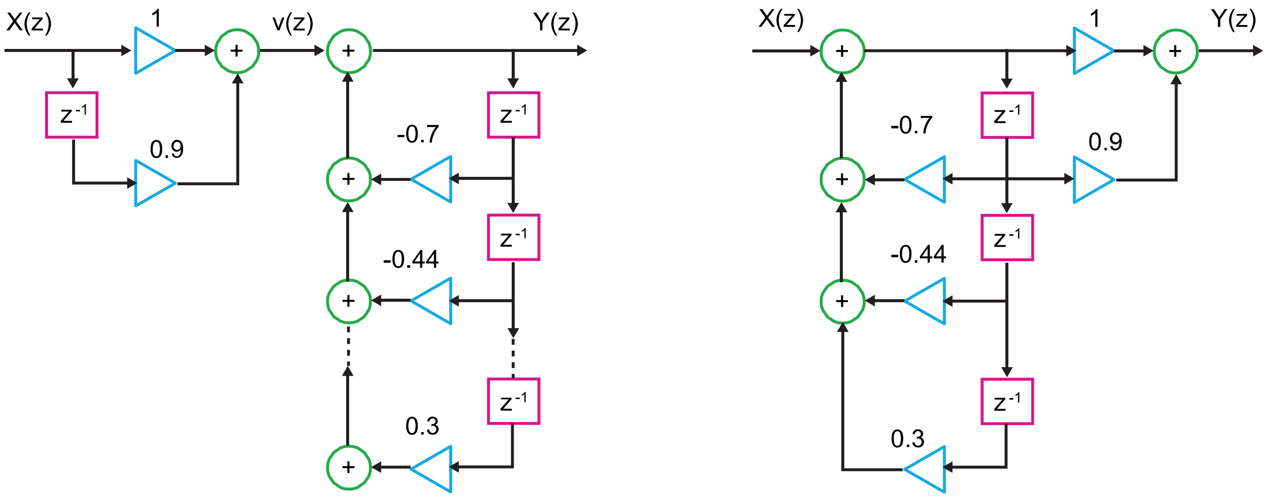
The step diagrams of straight form I and II are shown in the next figure.

(b) According to step diagrams (a) and (b), the number of computations in straight form I is:

- Multiplications: 5 for each output sample
- Additions: 4 for each output sample
- Delays: 4

and in straight form II is:

- Multiplications: 5 for each output sample
- Additions: 4 for each output sample
- Delays: 3



Step diagram: (a) Direct form I, (b) Direct form II

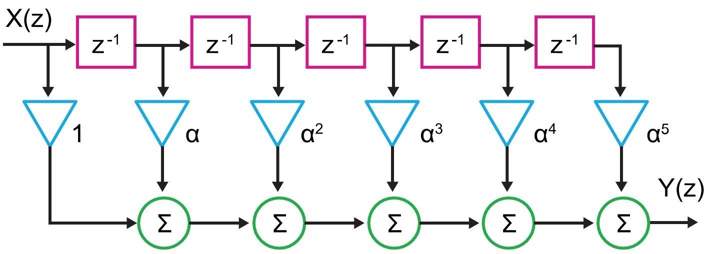
2. FIR filters

> **Example 2**
>
> (a) Draw the straight form of the FIR system with impulse response:
>
> $$h[n] = \begin{cases} a^n, & 0 \le n \le 5 \\ 0, & elsewhere \end{cases}$$
>
> (b) Calculate the number of multiplications and additions required to calculate each output sample as well as the number of delay registers.

Answer: (a) The impulse response is written:

$$h[n] = a^n\big[u[n] - n[n-6]\big] = \delta(0) + a\delta(1) + a^2\delta(2) + a^3\delta(3) + a^4\delta(4) + a^5\delta(5)$$

from which it follows that the straight form step diagram is:



FIR system structure in straight form (N=6)

(b) From the step diagram it follows that the number of calculations in the straight form is:

- Multiplications: 6 for each output sample
- Additions: 5 for each output sample
- Delays: 5

3. Lattice FIR Filters

Example 3

The reflection coefficients of a second-order FIR grating filter are $K_1 = 1/4$ and $K_2 = 1/8$. Find the transfer functions of prime $A_1(z)$ and second order $A_2(z)$, which connect the input $x[n]$ with $f_1[n]$ and $f_2[n]$, respectively.

Answer: We put $m = 1$in the relationship $A_m(z) = A_{m-1}(z) + K_m\, z^{-m} A_{m-1}(z^{-1})$and find:

$$A_1(z) = A_0(z) + K_1\, z^{-1} A_0(z^{-1}) \qquad (1)$$

The initial condition is $A_0(z) = 1$, therefore and $A_0(z^{-1}) = 1$. So relation (1) is calculated as:

$$A_1(z) = 1 + \frac{1}{4}\, z^{-1} \qquad (2)$$

From relation (2) we find that:

$$A_1(z^{-1}) = 1 + \frac{1}{4}\, z \qquad (3)$$

We put $m = 2$in the relation $A_m(z) = A_{m-1}(z) + K_m\, z^{-m} A_{m-1}(z^{-1})$and we have:

$$A_2(z) = A_1(z) + K_2\, z^{-2} A_1(z^{-1}) \qquad (4)$$

Substituting relations (2) and (3) into relation (4) we find:

$$A_2(z) = \left(1 + \frac{1}{4}\, z^{-1}\right) + \frac{1}{8} z^{-2} \left(1 + \frac{1}{4}\, z\right) = 1 + \frac{9}{32}\, z^{-1} + \frac{1}{8}\, z^{-2}$$

Example 4

To find the reflection coefficients of the second order FIR filter with transfer function:

$$A_2(z) = 1 - \frac{1}{2} z^{-2}$$

Answer: We put $m = 2$in the relationship$A_m(z) = A_{m-1}(z) + K_m\, z^{-m} A_{m-1}(z^{-1})$ and we have:

$$A_2(z) = A_1(z) + K_2\, z^{-2} A_1(z^{-1}) \qquad (1)$$

We put $m = 1$in the relation $A_m(z) = A_{m-1}(z) + K_m\, z^{-m} A_{m-1}(z^{-1})$and we have:

$$A_1(z) = A_0(z) + K_1\, z^{-1} A_0(z^{-1}) \qquad (2)$$

Since $A_0(z) = 1$and $A_0(z^{-1}) = 1$, relation 1 is written:

$$A_1(z) = 1 + K_1\, z^{-1} \qquad (3)$$

From relation (2) we find that:

$$A_1(z^{-1}) = 1 + K_1\, z \qquad (4)$$

We substitute relations (2), (3) and (4) into relation (1) and find:

$$A_2(z) = 1 + K_1 z^{-1} + K_2 z^{-2}(1 + K_1 z) = 1 + (K_1 + K_1 K_2) z^{-1} + K_2 z^{-2} \qquad (5)$$

We equate the corresponding coefficients of the given transfer function $A_2(z)$and relation

(5) and find:

$$K_2 = -\frac{1}{2}, \qquad K_1 = 0$$

4. Filters Lattice IIR

📖 Example 5

Convert the following IIR pole-zero filter to lattice form:

$$H(z) = \frac{0.25 + 0.5z^{-1} - 0.4z^{-2}}{1 - 0.1z^{-1} + z^{-2}}$$

Answer: First we will convert the denominator coefficients to reflection coefficients:

$$A_2(z) = 1 - 0.1z^{-1} + z^{-2}$$

also $m = 1$set$m = 2$ in the relationship$A_m(z) = A_{m-1}(z) + K_m\, z^{-m} A_{m-1}(z^{-1})$ and correspondingly we get:

$$A_2(z) = A_1(z) + K_2\, z^{-2} A_1(z^{-1}) \qquad (1)$$

$$A_1(z) = A_0(z) + K_1\, z^{-1} A_0(z^{-1}) \qquad (2)$$

Since $A_0(z) = 1$and $A_0(z^{-1}) = 1$, we find:

$$A_1(z) = 1 + K_1\, z^{-1} \qquad (3)$$

$$A_1(z^{-1}) = 1 + K_1\, z \qquad (4)$$

We substitute relations (2), (3) and (4) into relation (1) and find:

$$A_2(z) = 1 + K_1 z^{-1} + K_2 z^{-2}(1 + K_1 z) = 1 + (K_1 + K_1 K_2) z^{-1} + K_2 z^{-2} \qquad (5)$$

We equate the corresponding coefficients of the original function $A_2(z)$and relation (5) and find:

$$K_2 = 1, \qquad K_1 = -0.05$$

The coefficients $a_m$and $b_m$given in the pronunciation are:

$$a_m = \{\alpha_0, \alpha_1, \alpha_2\} = \{1, -0.1, 1\}$$

$$b_m = \{b_0, b_1, b_2\} = \{0.25, 0.5, -0.4\}$$

Finally, the coefficients $C_2$are calculated for$m = 2,1,0$ from the recursive relation:

$$y[n] = \sum_{m=0}^{M} C_m g_m[n]$$

and it is:

$$m = 2: \;\; C_2 = b_2 = -0.4$$

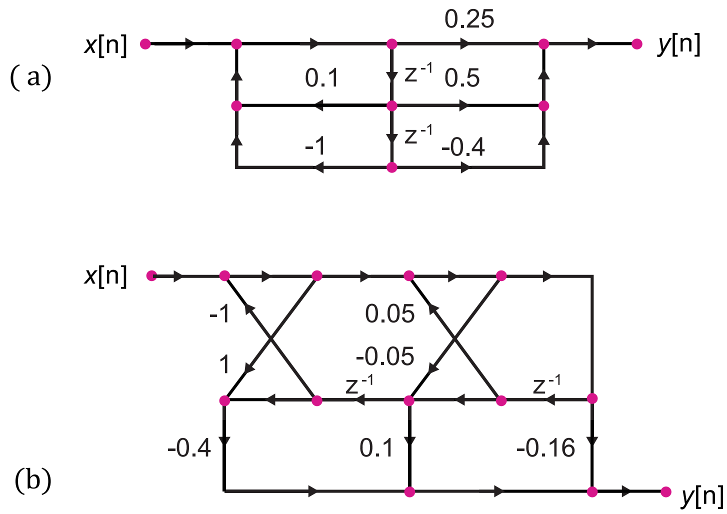
$$m = 1: \;\; C_1 = b_1 + C_2\, a_2[1] = 0.5 + (-0.4)\, 1 = 0.1$$

$$m = 0: \;\; C_0 = b_0 + C_1\, a_1[1] + C_2\, a_2[2] = 0.25 + 0.1\,(-0.1) + (-0.4)1 = -0.16$$

Therefore:

$$C_0 = -0.16, C_1 = 0.1, C_2 = -0.4$$

The figure below shows the straight form and the lattice form of the given IIR filter.



IIR filter: (a) Straight form, (b) Grid form