University of the Peloponnese
Electrical and Computer
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DIGITAL SIGNAL PROCESSING

Solved Examples

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SET #11 - Digital IIR Filters

- Design with direct pole-zero placement
- Invariant impulse response method
- Bilinear transform method

1. Design with direct pole-neutral placement

Example 1

Design an IIR bandpass filter with the following specifications:

- Passband centered at frequency: 3000 Hz
- Bandwidth of passband (3 dB): 1000 Hz
- Zero response at: 0 Hz and 5000 Hz
- Sampling frequency: 10000 Hz

Answer: Since zero amplitude of the frequency response at 0 Hz and 5000 Hz is desired, zeros should be placed at the corresponding points of the unit circle, i.e. at the points of the circle:

$$2\pi \frac{0\ Hz}{10000\ Hz} = 0\pi \text{ ή } 0^{o}$$

$$2\pi \frac{5000\ Hz}{10000\ Hz} = \pi \text{ ή } 180^{o}$$

Since the filter is required to have a passband centered at 3.000 Hz we will place a pole at the frequency:

$$2\pi \frac{3000\ Hz}{10000\ Hz} = \frac{3\pi}{5} \text{ ή } 108^{o}$$

and at a distance from the center of the unit circle:

$$r \approx 1 - \frac{\Delta f_{3dB}}{F_s}\pi = 1 - \frac{500}{10000}\pi = 1 - 0.05\pi = 0.95$$

For the coefficients of the transfer function to be real numbers, $H(z)$ the conjugate pole must also be placed in the appropriate position. So the equation (11.60) is:

$$H(z) = k\frac{(z-1)(z+1)}{(z-re^{j3\pi/5})(z-re^{-j3\pi/5})} = k\frac{z^2(1-z^{-2})}{z^2(1-2rz^{-1}\cos(3\pi/5)+r^2z^{-2})}$$
$$= k\frac{1-z^{-2}}{1+0.717125z^{-1}+0.9025\,z^{-2}}$$

The frequency response is:

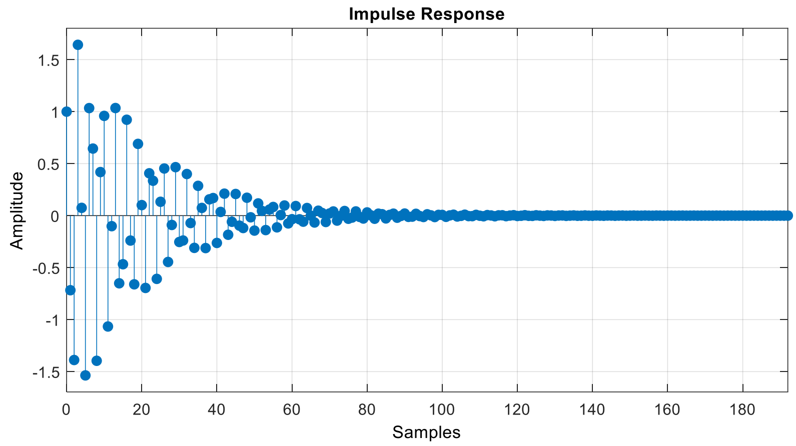
$$H(e^{j\omega}) = H(z)|_{z=e^{j\omega}} = \cdots = k\frac{1-e^{-2j\omega}}{1+0.717125e^{-j\omega}+0.9025\,e^{-2j\omega}}$$

For the transit zone:

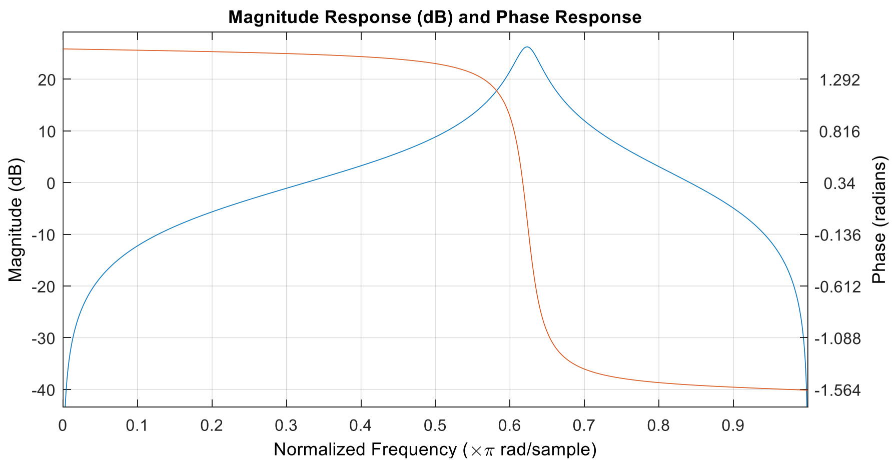
$$H\left(\frac{3\pi}{5}\right) = 1 \Rightarrow \ldots \Rightarrow k\frac{1.8090+j\,0.5878}{0.0483-j\,0.1516} = 1 \Rightarrow k = -0.0005-j\,0.0836$$

Based on the transfer function $H(z)$ we calculated and in order to draw the impulse response, the frequency response and the pole-zero diagram we write the following program in Matlab:

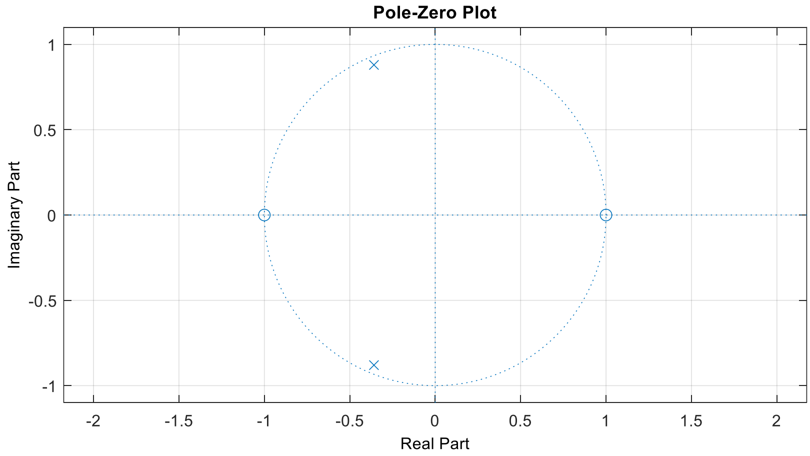
```
b = [1, 0, -1]; a = [1, 0.717125, 0.9025]; fvtool (b,a)
```



Impulse response



Frequency response (magnitude: blue color, phase: orange color)



Pole-zero diagram

2. Invariant impulse response method

Example 2

Transform the transfer function below $H(s)$ of the analog filter in a transfer function $H(z)$ of the digital filter.

$$H(s) = \frac{3s + 7}{s^2 + 4s + 3}$$

Answer: We write the $H(s)$ in fractional expansion:

$$H(s) = \frac{3s + 7}{s^2 + 4s + 3} = \frac{3s + 7}{(s + 1)(s + 3)} = \cdots = \frac{1}{s + 3} + \frac{2}{s + 1}$$

Its poles $H(s)$ are: $p_1 = -1$ and $p_2 = -3$. We also set $T_d = 0.1$ from the equation:

$$H(z) = \sum_{k=1}^{N} \frac{R_k}{1 - e^{p_k T_d} z^{-1}}$$

we find the transfer function of the digital filter:

$$H(z) = \frac{1}{1 - e^{-0.3} z^{-1}} + \frac{2}{1 - e^{-0.1} z^{-1}} = \frac{3 - 2.3865\, z^{-1}}{1 - 1.6457\, z^{-1} + 0.6703\, z^{-2}}$$

3. Bilinear Transform method

Example 3

Using the bilinear transform, plot the following points of the s plane on the z plane:

(α) $s_1 = -1 + j$ (β) $s_2 = 1 - j$ (γ) $s_3 = 2j$ (δ) $s_4 = -2j$

Answer: We value $T = 2$ the equation:

$$z = \frac{1 + (T/2)s}{1 - (T/2)s}$$

and we have:

(α) $z_1 = \frac{1+s_1}{1-s_1} = \frac{1-1+j}{1+1-j} = \frac{j}{2-j} = -0.2 + 0.4j = 0.447 \measuredangle\, 7.2^0$

since $|z_1| < 1$, the point z_1 lies inside the unit circle.

(β) $z_2 = \frac{1+s_1}{1-s_1} = \frac{1+1-j}{1-1+j} = \frac{2-j}{j} = -1 + 2j = 2.236 \measuredangle - 7.2^0$

since $|z_2| > 1$, the point z_2 is outside the unit circle.

(γ) $z_3 = \frac{1+s_1}{1-s_1} = \frac{1+2j}{1-2j} = -0.6 + 0.8j = 1\measuredangle\, 38.6^0$

since $|z_3| = 1$, the point z_3 lies on the positive half of the circumference of the unit circle.

(δ) $z_4 = \frac{1+s_1}{1-s_1} = \frac{1-2j}{1+2j} = \frac{2-j}{-j} = -0.2 - 0.8j = 1\measuredangle - 38.6^0$

since $|z_4| < 1$, the point z_4 lies on the negative half of the circumference of the unit circle.

📖Example 4

Using the bilinear transform to convert the analog transfer function filter to digital:

$$H(s) = \frac{3s+7}{s^2+4s+3}$$

Answer: We set a value $T = 2$ to the conversion ratio s ←→z so we have:

$$s = \frac{2}{T}\frac{z-1}{z+1}\bigg|_{T=2} = \frac{z-1}{z+1}$$

The required transfer function $H(z)$ is given by the equation:

$$H(z) = H(s)|_{s=\frac{z-1}{z+1}} = \frac{3\left(\frac{z-1}{z+1}\right)+7}{\left(\frac{z-1}{z+1}\right)^2 + 4\left(\frac{z-1}{z+1}\right)+3}$$

By simplifying we get:

$$H(z) = \frac{5z^2+7z+2}{4z^2+2z} = \frac{1.25 + 1.75z^{-1} + 0.5z^{-2}}{1+0.5z^{-1}}$$