University of the Peloponnese
Electrical and Computer Engineering Department

DIGITAL SIGNAL PROCESSING

Solved Examples

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SET #10 - Digital FIR Filters

- Introduction
- Window method
- Frequency Sampling Method

1. Introduction to digital filter design

Example 1

Let be $h[n]$the impulse response of an ideal low-pass filter with cutoff frequency ω_c. What type of ideal filter has a shock response $g[n] = (-1)^n h[n]$?

Answer: The frequency response $G(e^{j\omega})$ is:

$$G(e^{j\omega}) = \sum_{n=-\infty}^{\infty} g[n]e^{-jn\omega} = \sum_{n=-\infty}^{\infty} (-1)^n h[n]e^{-jn\omega}$$
$$= \sum_{n=-\infty}^{\infty} h[n]e^{-jn(\omega-\pi)} = H\left(e^{j(\omega-\pi)}\right)$$

Therefore, the function $G(e^{j\omega})$ is formed by shifting it $H(e^{j\omega})$ in frequency by π. Thus, if the passband of the ideal low-pass filter is $|\omega| \leq |\omega_c|$, then the passband of the frequency-responsive filter $G(e^{j\omega})$ will be $\pi - \omega_c < |\omega| \leq \pi$. Therefore, the impulse response filter $g(n)$ is a high-pass filter.

2. Window method

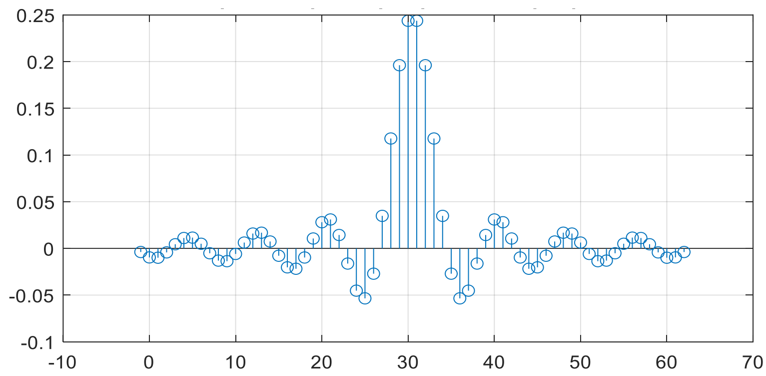
Example 2

Design a low-pass filter with cut-off frequency $\omega_c = 0.25\pi$ and length $N = 64$ coefficients, for all types of the above windows.

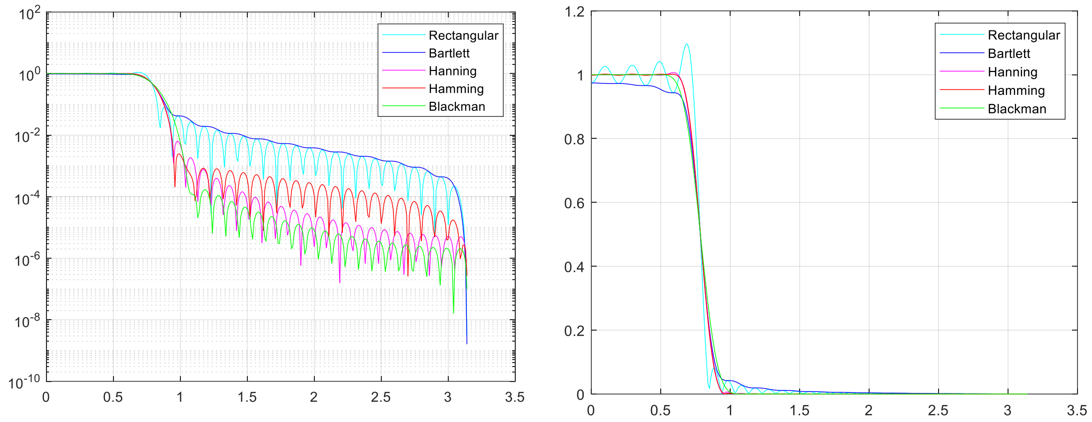
Answer: First we will calculate the impulse response of the ideal low-pass filter (LPF), by applying inverse DTFT to the frequency response $H_d(e^{j\omega})$, which for the case of the ideal low-pass filter is given by relation (13.17). We get the result:

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}\, d\omega = \frac{sin(\omega_c n)}{\pi n}$$

The impulse response of the ideal low-pass filter is:



Impulse response of an ideal low-pass filter (N =64)



Frequency response of filters ($N = 64$): (a) in semi-logarithmic scale, (b) in linear scale.

3. Frequency Sampling Method

Example 3

Design a low pass filter (LPF) with linear phase, cut-off frequency $\omega_c = 0.4\pi$ and length $N = 10$ coefficients.

Answer: We divide the frequency range [0, 2π] into N=10 equal-sized parts based on the relation $\omega_k = (2\pi/10)k = 0.2\pi k,\ \ k = 0,1,\dots,9$. The sampled frequency response $H[k]$ is shown in figure 13.11 (a).

$$k = 0, \quad \omega_0 = 0, \quad H[0] = 1$$

$$k = 1, \quad \omega_1 = 0.2\pi, \quad H[1] = 1$$

$$k = 2, \quad \omega_2 = 0.4\pi, \quad H[2] = 1$$

$$k = 3, \quad \omega_3 = 0.6\pi, \quad H[3] = 0$$

$$k = 4, \quad \omega_4 = 0.8\pi, \quad H[4] = 0$$

$$k = 5, \quad \omega_5 = \pi, \quad H[5] = 0$$

$$k = 6, \quad \omega_6 = 1.2\pi, \quad H[6] = 0$$

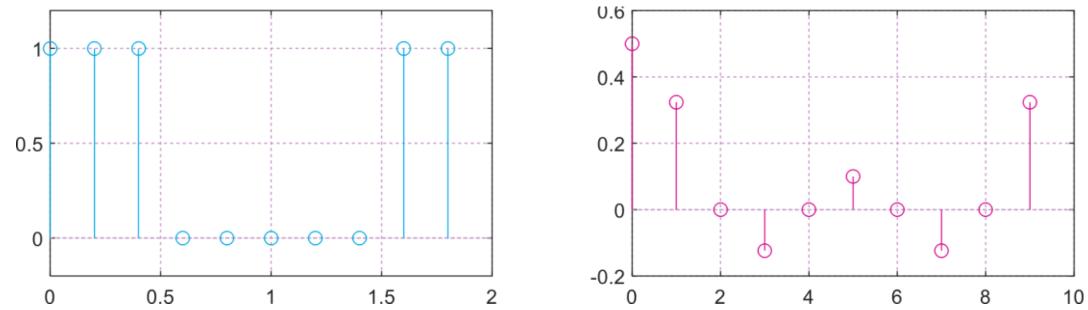
$$k = 7, \quad \omega_7 = 1.4\pi, \quad H[7] = 0$$

$$k = 8, \quad \omega_8 = 1.6\pi, \quad H[8] = 1$$

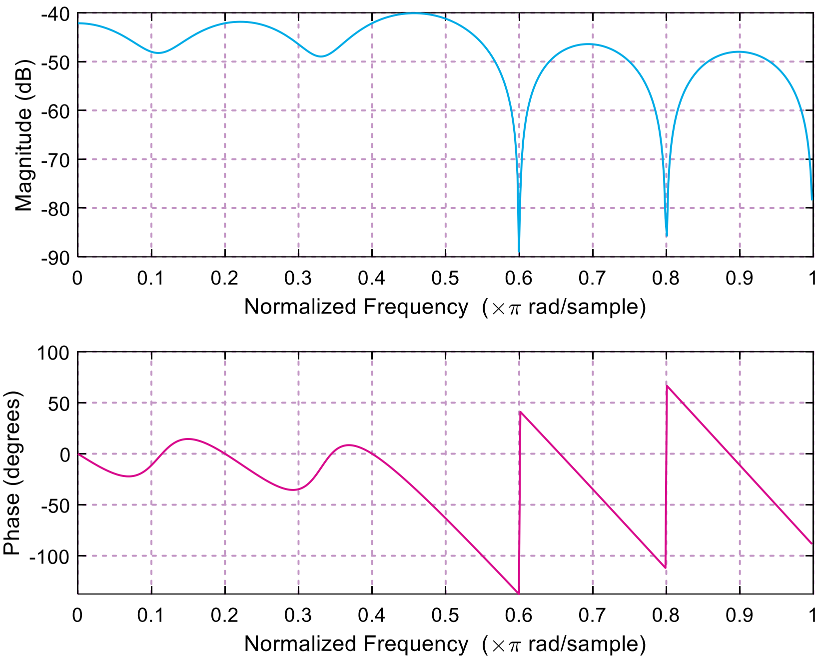
$$k = 9, \quad \omega_9 = 1.8\pi, \quad H[9] = 1$$

We can calculate the impulse response $h[n]$ of the approximation filter from relations (13.36) or (13.37). We choose (13.36) and for time values $0 \leq n < 9$ we have:

$$h[n] = \frac{1}{N}\sum_{k=0}^{N-1} H[k]\, e^{j2\pi nk/N} = \frac{1}{10}\sum_{k=0}^{9} H[k]\, e^{j2\pi nk/10}$$
$$= \frac{1}{10}\left(1e^{0} + 1e^{j2\pi n/10} + 1e^{j4\pi n/10} + 1e^{j16\pi n/10} + 1e^{j18\pi n/10}\right)$$



(a) Sampled frequency response (interval [0.2π])
and (b) Impulse response of approximating filter



Approximate filter frequency response $H\left(e^{j\omega}\right)$

The frequency response of the FIR approximation filter is shown in the figure. We observe the poor performance of the design as significant ripple occurs in the passband while the attenuation in the cutoff is only ~ 10dB. We can improve the performance of the method by increasing the order of the filter, however even for large values of the order there still remains ripple in the passband and little attenuation in the cutoff. An improvement of the method is based on the addition of samples in the transition zone.