



DIGITAL SIGNAL PROCESSING

Solved Examples
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SET #4 – Study of Discrete-Time Systems with Difference Equations

- Recursive and Non-Recursive Discrete Time Systems
- Homogeneous and Partial Solution
- Study of Asymptotic Stability of GACM Discrete Time System

1. Recursive and Non-Recursive Discrete Time Systems

Example 1

A discrete-time recursive (ARMA) system is described by the first-order Linear Difference Equation with Constant Coefficients (LDECC):

$$y[n] = -\frac{1}{2}y[n-1] + x[n] - x[n-1]$$

(a) Calculate the output of the system when it is at initial rest and receives input: (i) $x[n] = u[n]$ and (ii) $x[n] = 2u[n]$.

(b) Repeat the solution for the same inputs, but for an initial condition $y[-1] = -2$.

In both cases, investigate whether the system has a linear behavior or not.

Answer: (a- i) We apply input $x_1[n] = u[n]$ and iteratively solve the LDECC for zero initial conditions and for values of $n = 0, 1, 2, \dots$

$$n = 0, y_1[0] = -0.5 y_1[-1] + u[0] - u[-1] = 0 + 1 - 0 \Rightarrow y_1[0] = 1$$

$$n = 1, y_1[1] = -0.5 y_1[0] + u[1] - u[0] = -0.5 + 1 - 1 \Rightarrow y_1[1] = -0.5$$

$$n = 2, y_1[2] = -0.5 y_1[1] + u[2] - u[1] = 0.25 + 1 - 1 \Rightarrow y_1[2] = 0.25$$

$$n = 3, y_1[3] = -0.5 y_1[2] + u[3] - u[2] = -0.125 + 1 - 1 \Rightarrow y_1[3] = -0.125$$

$$n = 4, y_1[4] = -0.5 y_1[3] + u[4] - u[3] = 0.0625 + 1 - 1 \Rightarrow y_1[4] = 0.0625$$

Note: Valid $y_1[-1] = 0$ because the system is in an initial quiescent state.

(a- ii) We repeat the solution for input $x_2[n] = 2u[n]$ and zero initial conditions:

$$n = 0, y_2[0] = -0.5 y_2[-1] + 2u[0] - 2u[-1] = 0 + 2 - 0 \Rightarrow y_2[0] = 2$$

$$n = 1, y_2[1] = -0.5 y_2[0] + 2u[1] - 2u[0] = -1 + 2 - 2 \Rightarrow y_2[1] = -1$$

$$n = 2, y_2[2] = -0.5 y_2[1] + 2u[2] - 2u[1] = 0.5 + 2 - 2 \Rightarrow y_2[2] = 0.5$$

$$n = 3, y_2[3] = -0.5 y_2[2] + 2u[3] - 2u[2] = -0.25 + 2 - 2 \Rightarrow y_2[3] = -0.25$$

$$n = 4, y_2[4] = -0.5 y_2[3] + 2u[4] - 2u[3] = 0.125 + 2 - 2 \Rightarrow y_2[4] = 0.125$$

We notice that doubling the input, i.e., $x_2[n] = 2x_1[n]$, results in a doubling of the output, i.e. $y_2[n] = 2y_1[n]$. Therefore, for zero initial conditions the system has linear behavior.

(b- i) We apply input $x_1[n] = u[n]$ and iteratively solve the LDECC for initial condition $y[-1] = -2$ and values of $n = 0,1,2, \dots$

$$n = 0, y_1[0] = -0.5 y_1[-1] + u[0] - u[-1] = 1 + 1 - 0 \Rightarrow y_1[0] = 2$$

$$n = 1, y_1[1] = -0.5 y_1[0] + u[1] - u[0] = -1 + 1 - 1 \Rightarrow y_1[1] = -1$$

$$n = 2, y_1[2] = -0.5 y_1[1] + u[2] - u[1] = 0.5 + 1 - 1 \Rightarrow y_1[2] = 0.5$$

$$n = 3, y_1[3] = -0.5 y_1[2] + u[3] - u[2] = -0.25 + 1 - 1 \Rightarrow y_1[3] = -0.25$$

$$n = 4, y_1[4] = -0.5 y_1[3] + u[4] - u[3] = 0.125 + 1 - 1 \Rightarrow y_1[4] = 0.125$$

(b- ii) We repeat the solution for input $x_2[n] = 2u[n]$ and initial condition $y[-1] = -2$:

$$n = 0, y_2[0] = -0.5 y_2[-1] + 2u[0] - 2u[-1] = 1 + 2 - 0 \Rightarrow y_2[0] = 3$$

$$n = 1, y_2[1] = -0.5 y_2[0] + 2u[1] - 2u[0] = -1.5 + 2 - 2 \Rightarrow y_2[1] = -1.5$$

$$n = 2, y_2[2] = -0.5 y_2[1] + 2u[2] - 2u[1] = 0.75 + 2 - 2 \Rightarrow y_2[2] = 0.75$$

$$n = 3, y_2[3] = -0.5 y_2[2] + 2u[3] - 2u[2] = -0.375 + 2 - 2 \Rightarrow y_2[3] = -0.375$$

$$n = 4, y_2[4] = -0.5 y_2[3] + 2u[4] - 2u[3] = 0.1875 + 2 - 2 \Rightarrow y_2[4] = 0.1875$$

We notice that doubling the input, i.e. $x_2[n] = 2x_1[n]$, no doubling of the output, i.e. $y_2[n] = 2y_1[n]$. Therefore, the system does not have a linear behavior and this is due to the non-zero initial conditions.

Example 2

To calculate the impact response of the non-retrograde (FIR) (Moving Average, MA) of the system, which is in a state of initial rest and is described by the first-order LDECC:

$$y[n] = \frac{1}{2}(x[n] + x[n - 1])$$

Check the system for linearity and time invariance.

Answer: As long as the impulse response is requested, $h[n]$ we consider it an entry $x[n] = \delta[n]$. We iteratively solve the LDECC for values of $n = 0,1,2, \dots$

$$n = 0, \quad h[0] = 0.5(\delta[0] - \delta[-1]) = 0.5(1 - 0) \Rightarrow h[0] = 0.5$$

$$n = 1, \quad h[1] = 0.5(\delta[1] - \delta[0]) = 0.5(0 - 1) \Rightarrow h[1] = -0.5$$

$$n = 2, \quad h[2] = 0.5(\delta[2] - \delta[1]) = 0.5(0 - 0) \Rightarrow h[2] = 0$$

$$n = 3, \quad h[3] = 0.5(\delta[3] - \delta[2]) = 0.5(0 - 0) \Rightarrow h[3] = 0$$

$$n = 4, \quad h[4] = 0.5(\delta[4] - \delta[3]) = 0.5(0 - 0) \Rightarrow h[4] = 0$$

From LDECC we observe that each output sample is obtained as an average of the current and the previous input sample, i.e. the output signal is the smoothed input signal.

Linearity check: If we put in the system a combined input $x[n] = ax_1[n] + bx_2[n]$ the output is:

$$\begin{aligned} y[n] &= \frac{1}{2}(ax_1[n] + bx_2[n] + ax_1[n-1] + bx_2[n-1]) \\ &= \frac{1}{2}(ax_1[n] + ax_1[n-1]) + \frac{1}{2}(bx_2[n] + bx_2[n-1]) \\ &= ay_1[n] + by_2[n] \end{aligned}$$

We observe that a linearly combined output results, so the system is linear.

Time variability control: If we put a time-shifted input in the system, $x[n] = x[n - n_0]$ the output is:

$$y'[n] = \frac{1}{2}(x[n - n_0] + x[n - n_0 - 1]) = y[n - n_0]$$

We notice that a respectively time-shifted output results $y[n - n_0]$, so the system is time-invariant.

2. Homogeneous and Partial Solution

In the following examples we will deal with the calculation of the impulse response when the LDECC is known and the system is initially relaxed, i.e. it is Linear and Shift Invariant (LSI). Since we are looking for the impulse response $h[n]$, we put in LDECC as input $x[n] = \delta[n]$ and zero initial conditions, because the system is initially relaxed. But the appearance at the entrance of such an active sequence as Delta creates new initial conditions in the system. The output in this case, i.e. the impulse response, will result from the solution of the homogeneous equation (zero-input response) for the new initial conditions. Therefore, the problem of finding the impulse response from the LDECC is reduced to finding the new initial conditions due to the Delta sequence and then solving the homogeneous solution to find the zero-input response, i.e. the impulse response.

Example 3

Find the impulse response of the recursive system described by LDECC:

$$y[n] + \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n]$$

Answer: We set as input $x[n] = \delta[n]$ and zero initial conditions because the system is in a state of initial rest. We have:

$$h[n] + \frac{3}{4}h[n-1] + \frac{1}{8}h[n-2] = \delta[n]$$

The homogeneous equation is:

$$h[n] + \frac{3}{4}h[n-1] + \frac{1}{8}h[n-2] = 0$$

The characteristic polynomial is:

$$\lambda^2 + \frac{3}{4}\lambda + \frac{1}{8}$$

and has roots $\lambda_1 = -1/2$ and $\lambda_2 = -1/4$. Therefore, the solution of the homogeneous equation (zero input response) is:

$$h[n] = A_1\lambda_1^n + A_2\lambda_2^n = A_1\left(-\frac{1}{2}\right)^n + A_2\left(-\frac{1}{4}\right)^n, \quad n \geq 0$$

We will calculate the values of the constants A_1 and A_2 using the initial conditions. For the given LDECC apply:

$$h[0] + \frac{3}{4}h[-1] + \frac{1}{8}h[-2] = \delta[0] \Rightarrow h[0] + 0 + 0 = 1 \Rightarrow h[0] = 1$$

and

$$h[1] + \frac{3}{4}h[0] + \frac{1}{8}h[-1] = \delta[1] \Rightarrow h[1] + \frac{3}{4} + 0 = 0 \Rightarrow h[1] = -\frac{3}{4}$$

Applying these initial conditions to the homogeneous solution so far, we have:

$$h[0] = A_1\left(-\frac{1}{2}\right)^0 + A_2\left(-\frac{1}{4}\right)^0 \Rightarrow A_1 + A_2 = 1$$

and

$$h[1] = A_1\left(-\frac{1}{2}\right)^1 + A_2\left(-\frac{1}{4}\right)^1 \Rightarrow -\frac{1}{2}A_1 - \frac{1}{4}A_2 = -\frac{3}{4}$$

We write the pair of equations with unknowns A_1 and A_2 in tabular form:

$$\begin{bmatrix} 1 & 1 \\ -1/2 & -1/4 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -3/4 \end{bmatrix}$$

We solve the system and find: $A_1 = 1/2$ and $A_2 = 2$.

Since the right-hand member of the LDECC is $x[n]$ it follows that the impulse response is equal to the homogeneous solution, that is:

$$h_g[n] = h[n] = \left[\frac{1}{2}\left(-\frac{1}{2}\right)^n + 2\left(-\frac{1}{4}\right)^n \right] u[n]$$

Example 4

Find the impulse response of the recursive system described by LDECC:

$$y[n] - \frac{2}{15}y[n-1] - \frac{1}{15}y[n-2] = x[n] + 2x[n-1]$$

Answer: We apply the same solution methodology. We also set zero initial conditions as input $x[n] = \delta[n]$ because the system is initially relaxed. We have:

$$h[n] - \frac{2}{15}h[n-1] - \frac{1}{15}h[n-2] = \delta[n]$$

The homogeneous equation is:

$$h[n] - \frac{2}{15}h[n-1] - \frac{1}{15}h[n-2] = 0$$

The characteristic polynomial is:

$$\lambda^2 - \frac{2}{15}\lambda - \frac{1}{15}$$

and has roots $\lambda_1 = 1/3$ and $\lambda_2 = -1/5$. Therefore, the solution of the homogeneous equation (zero input response) is:

$$h[n] = A_1\lambda_1^n + A_2\lambda_2^n = A_1\left(\frac{1}{3}\right)^n + A_2\left(-\frac{1}{5}\right)^n, \quad n \geq 0$$

We will calculate the values of the constants A_1 and A_2 using the initial conditions. For the given LDECC apply:

$$h[0] - \frac{2}{15}h[-1] - \frac{1}{15}h[-2] = \delta[0] + 2\delta[-1] \Rightarrow h[0] + 0 + 0 = 1 + 0 \Rightarrow h[0] = 1$$

and

$$h[1] - \frac{2}{15}h[0] - \frac{1}{15}h[-1] = \delta[1] + 2\delta[0] \Rightarrow h[1] - \frac{2}{15} - 0 = 0 + 2 \Rightarrow h[1] = \frac{32}{15}$$

Applying these initial conditions to the homogeneous solution so far, we have:

$$h[0] = A_1\left(\frac{1}{3}\right)^0 + A_2\left(-\frac{1}{5}\right)^0 \Rightarrow A_1 + A_2 = 1$$

and

$$h[1] = A_1\left(\frac{1}{3}\right)^1 + A_2\left(-\frac{1}{5}\right)^1 \Rightarrow \frac{1}{3}A_1 - \frac{1}{5}A_2 = \frac{32}{15}$$

We write the pair of equations with unknowns A_1 and A_2 in tabular form:

$$\begin{bmatrix} 1 & 1 \\ 1/3 & -1/5 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 32/15 \end{bmatrix}$$

We solve the system and find: $A_1 = 35/8$ and $A_2 = -27/8$.

So the homogeneous solution is:

$$h[n] = \frac{1}{8} \left[35 \left(\frac{1}{3}\right)^n - 27 \left(-\frac{1}{5}\right)^n \right] u[n]$$

Because the right-hand member of the LDECC is $x[n] + 2x[n-1]$ it follows that the impulse response is equal to the sum $h[n] + 2h[n-1]$, that is:

$$\begin{aligned} h_g[n] &= h[n] + 2h[n-1] \\ &= \frac{1}{8} \left[35 \left(\frac{1}{3}\right)^n - 27 \left(-\frac{1}{5}\right)^n \right] u[n] + \frac{1}{4} \left[35 \left(\frac{1}{3}\right)^{n-1} - 27 \left(-\frac{1}{5}\right)^{n-1} \right] u[n-1] \end{aligned}$$

Example 5

Find the impulse response of the system with initial conditions $y[-1] = y[-2] = 1/8$ described by the LDECC:

$$y[n] - 8y[n-1] + 16y[n-2] = x[n] - 2x[n-1]$$

Answer: Finding a partial solution: For input $x[n] = \delta[n]$, the partial/zero-state solution according to Table 8.2 is:

$$y_p[n] = 0, \quad n \geq 0$$

Finding a homogeneous solution:

- Descriptive polynomial: $\lambda^2 - 8\lambda + 16$
- Natural frequencies: $\lambda_{1,2} = \frac{1}{2} (1 \pm j\sqrt{3}) = e^{\pm j\pi/3}$ (imaginary conjugates)
- Homogeneous solution/zero input: $y_h[n] = A_1 e^{jn\pi/3} + A_2 e^{-jn\pi/3}$
- General solution: $y[n] = A_1 e^{jn\pi/3} + A_2 e^{-jn\pi/3}, n \geq 0$ (1)

To calculate the constants A_1 and A_2 , we solve the LDECC for $n = 0$ and $n = 1$ and we have:

$$y[0] = 8y[-1] - 16y[-2] + x[0] - 2x[-1] = 1 - 2 + 1 - 0 = 0 \Rightarrow y[0] = 0$$

$$y[1] = 8y[0] - 16y[-1] + x[1] - 2x[0] = 0 - 2 + 0 - 2 \Rightarrow y[1] = 4$$

We substitute the new initial conditions into relation (1) and find:

$$y[0] = y_h[0] = A_1 e^0 + A_2 e^0 = A_1 + A_2 = 0$$

$$y[1] = y_h[1] = A_1 e^{j\pi/3} + A_2 e^{-j\pi/3} = 4$$

We write the pair of equations with unknowns A_1 and A_2 in tabular form:

$$\begin{bmatrix} 1 & 1 \\ e^{j\pi/3} & e^{-j\pi/3} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Solving the system, we find:

$$A_1 = 0 - 2.31j = 2.31 e^{-j\pi/2}$$

$$A_2 = 0 + 2.31j = 2.31 e^{j\pi/2}$$

Substituting the constants A_1 and A_2 in (1) and using the Euler relation we find:

$$\begin{aligned} h[n] &= 2.31 (e^{jn\pi/3} e^{-j\pi/2} + 2.31 e^{-jn\pi/3} e^{j\pi/2}) u[n] = 2.31 \left(e^{j\pi(\frac{n}{3}-\frac{1}{2})} + e^{-j\pi(\frac{n}{3}-\frac{1}{2})} \right) u[n] \\ &= 4,62 \sin\left(\frac{\pi n}{3}\right) u[n] \end{aligned}$$

Since the right-hand member of the LDECC is $x[n] + 2x[n-1]$, the impulse response is equal to the sum $h[n] + 2h[n-1]$, i.e. it is:

$$h_g[n] = h[n] + 2h[n-1] = 4,62 \left[\sin\left(\frac{\pi n}{3}\right) + 2 \sin\left(\frac{\pi(n-1)}{3}\right) \right] u[n]$$

Example 6

Calculate the homogeneous solution (zero-input response), the partial solution (zero-state response), and the total output for input $x[n] = (-0.6)^n u[n]$ for a recursive system with initial conditions $y[-1] = 1$ and $y[-2] = 1$ and LDECC:

$$y[n] - \frac{3}{2}y[n-1] - y[n-2] = x[n]$$

Answer: Finding a partial solution: For input $x[n] = (-0.6)^n u[n]$, the partial solution (zero-state response) according to Table 8.2 is:

$$y_p[n] = C_1 (-0.6)^n u[n]$$

The partial solution satisfies the LDECC, namely:

$$C_1 (-0.6)^n u[n] - \frac{3}{2} C_1 (-0.6)^{n-1} u[n-1] - C_1 (-0.6)^{n-2} u[n-2] = (-0.6)^n u[n]$$

We find its value C_1 by putting $n = 2$ in the above equation, so we have:

$$C_1(-0.6)^2 u[2] - \frac{3}{2} C_1(-0.6)^1 u[1] - C_1(-0.6)^0 u[0] = (-0.6)^2 u[2]$$

from which we find:

$$C_1 = \frac{18}{13}$$

So the partial solution is:

$$y_p[n] = \frac{18}{13}(-0.6)^n u[n]$$

Finding a homogeneous solution: The characteristic polynomial is:

$$\lambda^2 - \frac{3}{2}\lambda - 1$$

and the natural frequencies are:

$$\lambda_1 = -\frac{1}{2} \quad \text{και} \quad \lambda_2 = 2$$

Therefore, the homogeneous solution (zero-input response) is:

$$y_h[n] = A_1 \left(-\frac{1}{2}\right)^n + A_2(2)^n$$

The total solution is the sum of the partial and homogeneous solutions:

$$y[n] = y_p[n] + y_h[n] = \frac{18}{13}(-0.6)^n u[n] + A_1 \left(-\frac{1}{2}\right)^n + A_2(2)^n$$

To calculate the constants A_1 and A_2 , we solve the LDECC for $n = 0$ and $n = 1$ and we have:

$$y[0] = \frac{3}{2}y[-1] - y[-2] + x[0] = \frac{3}{2} - 1 + 1 \Rightarrow y[0] = \frac{3}{2}$$

$$y[1] = \frac{3}{2}y[0] - y[-1] + x[1] = \frac{9}{4} - 1 + (-0.6) \Rightarrow y[1] = \frac{13}{20}$$

We substitute the new initial conditions into the complete solution and find:

$$y[0] = y_p[0] + y_h[0] = \frac{3}{2} \Rightarrow \dots \Rightarrow A_1 + A_2 = \frac{3}{26}$$

$$y[1] = y_p[1] + y_h[1] = \frac{13}{20} \Rightarrow \dots \Rightarrow -\frac{1}{2}A_1 + 2A_2 = -\frac{47}{260}$$

We solve the system and find $A_1 = 0,1646$ and $A_2 = -0,0492$.

$$y[n] = \frac{18}{13}(-0.6)^n u[n] + 0,1646 \left(-\frac{1}{2}\right)^n - 0,0492(2)^n$$

Example 7

Find the impulse response of the recursive system described by LDECC $y[n] + 0.1y[n - 1] - 0.72y[n - 2] = x[n]$ and is initially relaxed. Verify the solution using Matlab / Octave code.

Answer: We apply the same solution methodology. We also set zero initial conditions as

input $x[n] = \delta[n]$ because the system is at initial rest.

- Characteristic polynomial: $\lambda^2 + 0.1\lambda - 0.72 = (\lambda - 0.8)(\lambda + 0.9)$ (1)
- Natural frequencies: $\lambda_1 = 0.8$ and $\lambda_2 = -0.9$ (simple roots)
- Homogeneous solution/zero input: $y_h[n] = A_1(0.8)^n + A_2(-0.9)^n, n \geq 0$ (2)
- Partial solution/null state: $y_p[n] = 0$, because $x[n] = \delta[n]$
- Complete solution: $y[n] = y_h[n] + y_p[n] = y_h[n]$ (3)

Finding a homogeneous solution: Since the system is initially relaxed, the initial conditions are zero, that is: $y[-1] = y[-2] = 0$. Solving the LDECC for $n = 0$ and $n = 1$, we have for the complete solution the new initial conditions due to the application of the impulse input:

$$y[0] = -0.1y[-1] + 0.72y[-2] + x[0] = 1 \Rightarrow y[0] = 1$$

$$y[1] = -0.1y[0] + 0.72y[-1] + x[1] = -0.1 \Rightarrow y[1] = -0.1$$

To calculate the constants A_1 and A_2 , we solve the homogeneous solution (2) for $n = 0$ and $n = 1$, consider equation (3) and find:

$$y[0] = y_h[0] = A_1(0.8)^0 + A_2(-0.9)^0 \Rightarrow A_1 + A_2 = 1 \quad (4)$$

$$y[1] = y_h[1] = A_1(0.8)^1 + A_2(-0.9)^1 \Rightarrow 0.8A_1 - 0.9A_2 = -0.1 \quad (5)$$

We solve the above system of equations (4) and (5) in terms of A_1, A_2 and find:

$$A_1 = \frac{8}{17} \quad \text{and} \quad A_2 = \frac{9}{17}$$

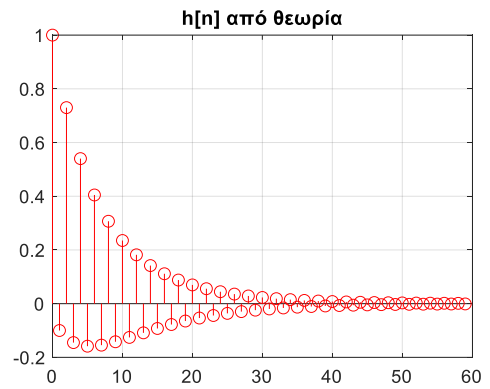
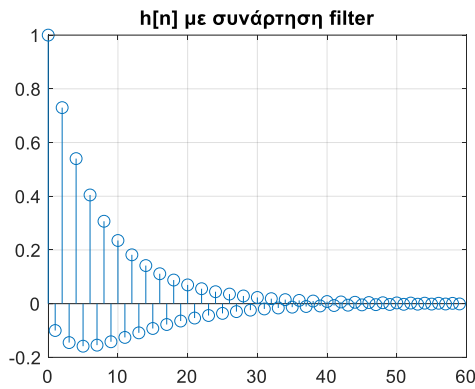
Therefore, the complete solution of the LDECC is:

$$y[n] = y_h[n] = \left[\frac{8}{17}(0.8)^n + \frac{9}{17}(-0.9)^n \right] u[n]$$

Based on the right-hand member of the LDECC, the impulse response is equal to the homogeneous solution and is:

$$h[n] = y_h[n] = \left[\frac{8}{17}(0.8)^n + \frac{9}{17}(-0.9)^n \right] u[n]$$

Matlab / Octave for the solution of LDECC uses the function **filter (b, a, x)** where a, b are matrices with the coefficients $a[k]$ and $b[m]$ and **x** is the input signal. The function **filtic (b, a, y, x)** creates the initial conditions for the **filter()** function. Specifically, it converts past input x and output y to initial input and output conditions, respectively. Figure (a) shows the impulse response as calculated with the **filter()** function and in figure (b) as derived from the theoretical solution presented above. We observe that the theoretical solution coincides with the computational solution.



filter () function

(b) Impulse response calculated from theoretical solution

3. Study of Asymptotic Stability of LTI Discrete Time System

Example 7

Investigate the asymptotic stability of the discrete-time system with LDECC:

$$9y[n] + 13y[n - 1] + 5y[n - 2] + y[n - 3] = x[n] + 2x[n - 1]$$

Answer: The expatriate LDECC is:

$$9y[n] + 13y[n - 1] + 5y[n - 2] + y[n - 3] = 0$$

from which it follows that the characteristic equation is:

$$\begin{aligned} \lambda^3 + 4\lambda^2 + 9\lambda + \lambda^2 + 4\lambda + 9 &= (\lambda + 1)(\lambda^2 + 4\lambda + 9) \\ &= (\lambda + 1)(\lambda + 2 - 3j)(\lambda + 2 + 3j) = 0 \end{aligned}$$

Therefore, the roots (natural frequencies) are $\lambda_1 = -1$, $\lambda_2 = -2 + 3j$, $\lambda_3 = -2 - 3j$. The measures of the roots are: $|\lambda_1| = 1$, $|\lambda_2| = |\lambda_3| = \sqrt{13}$. Since there are roots outside the unit circle, we conclude that the system is asymptotically unstable.