



University of the Peloponnese

Electrical and Computer  
Engineering Department

# Digital Signal Processing

## Unit 11: Implementation Structures of Discrete Time Systems

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# Infinite Impulse Response (IIR) Filters

- Direct Form I
- Direct Form II
- Cascade form
- Parallel Form

# Description of IIR Systems

Time Domain:

$$y[n] = \sum_{m=0}^M b_m x[n-m] - \sum_{k=1}^N a_k y[n-k] \quad (1)$$

Frequency Domain:

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} = \frac{B(z)}{A(z)} = \frac{\sum_{m=0}^M b_m z^{-m}}{1 + \sum_{k=1}^N a_k z^{-k}} \\ &= \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}} \end{aligned} \quad (2)$$

The **order** of the system is  $N$  though  $a[N] \neq 0$  it is the one that determines the number of delay units for the system implementation. This count is important because it costs memory locations.

# Direct Form I

- The LDECC of equation (1) can be split into two equations as follows:

$$v[n] = \sum_{m=0}^M b_m x[n - m]$$

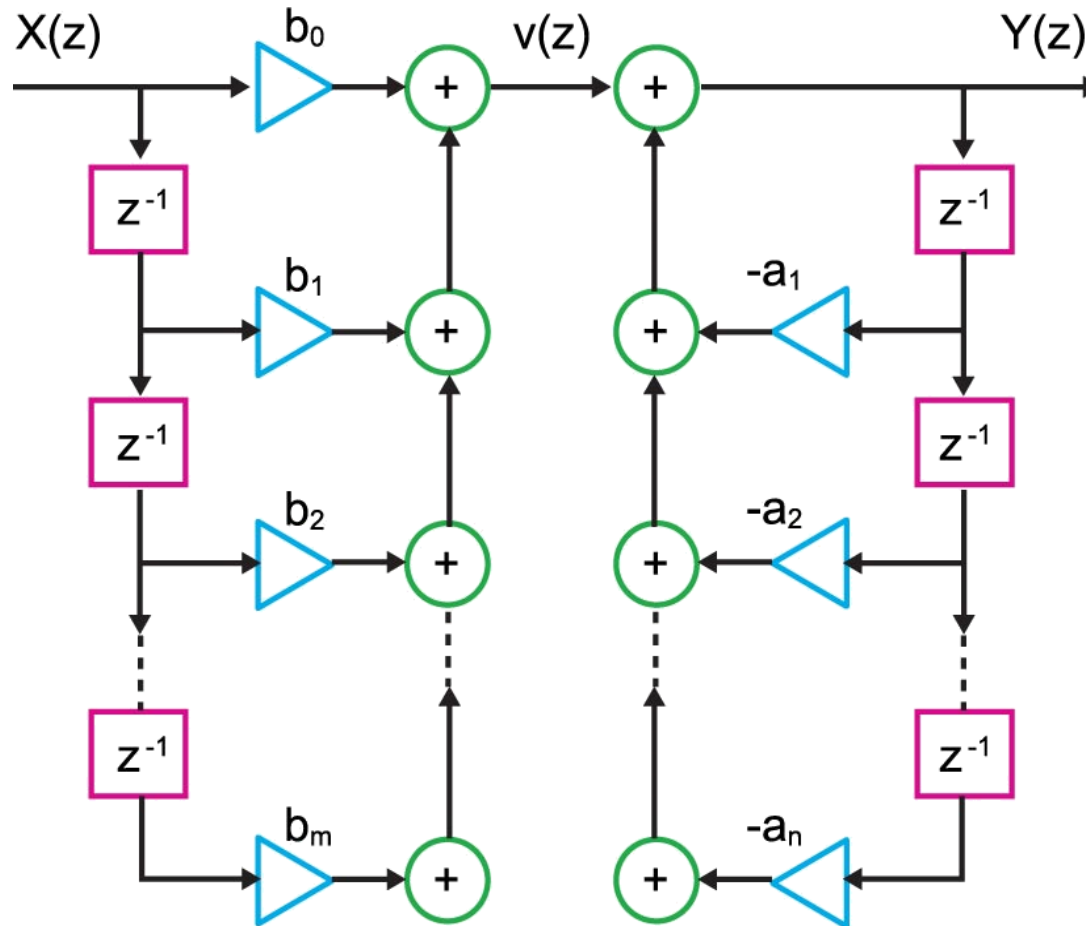
$$y[n] = v[n] - \sum_{k=1}^N a_k y[n - k]$$

- The first equation describes a subsystem with input signal  $x[n]$  and output  $v[n]$ . From the form of the equation it follows that this subsystem is all-pole **only**.
- The second equation describes an FIR subsystem that accepts the signal as input  $v[n]$  and produces the output  $y[n]$  of the overall system. This subsystem implements system zeros.
- The two subsystems are serial connected and are described in the frequency domain by the following equations:

$$V(z) = [b_0 + b_1 z^{-1} + \dots + b_M z^{-M}] X(z)$$

$$Y(z) = V(z) - [a_1 z^{-1} + \dots + a_N z^{-N}] Y(z)$$

# Direct Form I



IIR system structure in Direct form I

# Direct Form I

- Subsystem is implemented as a **tapped delay line**
- For an IIR system of order  $N$ , the total number of delay units is  $M + N$ (when  $M \leq N$ ) or  $2N$ (when  $M = N$ ).
- From the figure it follows that the number of calculations in the Direct form I is:
  - Multiplications:  $M + N + 1$ for each output sample
  - Additions:  $M + N$ for each output sample
  - Delays:  $M + N$

# Direct Form II

- Based on the commutative property, we mutually interchange the order of the subsystems of the Direct form I, so the LDECC is written:

$$w[n] = x[n] - \sum_{k=1}^N a_k w[n - k]$$

$$y[n] = \sum_{m=0}^M b_m w[n - m]$$

- The above equations in the frequency domain are written:

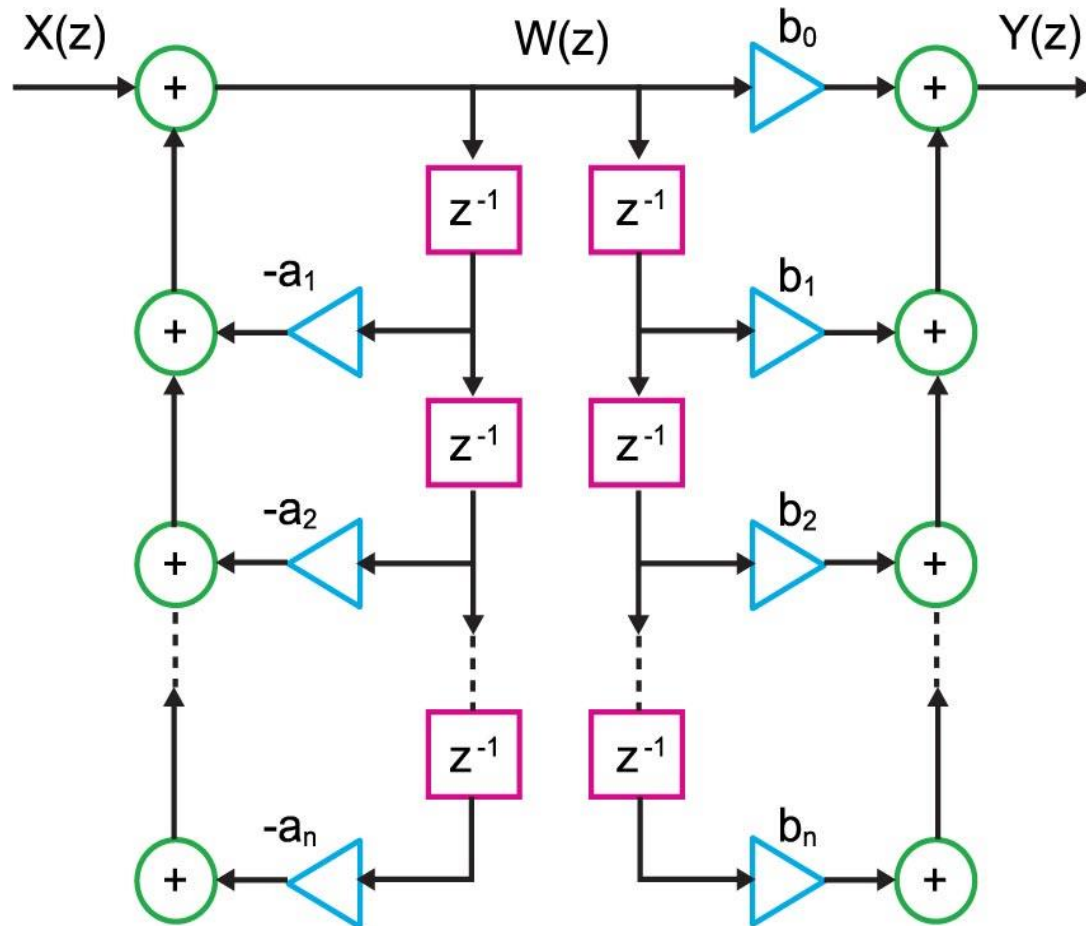
$$W(z) = X(z) - [a_1 z^{-1} + \dots + a_N z^{-N}]W(z)$$

$$Y(z) = [b_0 + b_1 z^{-1} + \dots + b_M z^{-M}]W(z)$$

- With this process, first the zeros and then the poles of the system are realized.



# Direct Form II

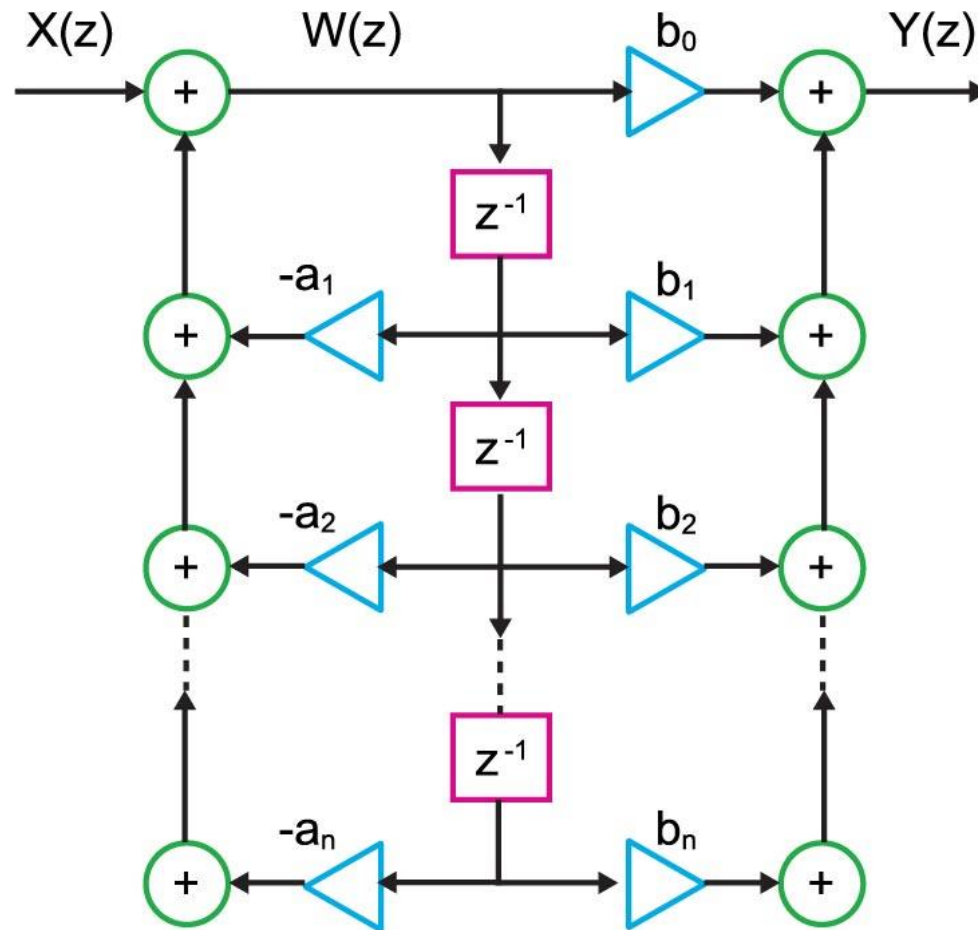


Intermediate description of IIR system, considering that  $M = N$

# Direct Form II

- Because the delay units have the same input  $W(z)$  we can erase the one delay line, so the number of delay units from  $2N$ , will be reduced to  $N$ .
- The resulting stage diagram is called **Direct Form II** or **normal** because the minimum amount of delay units is used and is shown in the next figure.
- From the figure it follows that the number of calculations in the Direct form II is:
  - Multiplications:  $M + N + 1$  for each output sample
  - Additions:  $M + N$  for each output sample
  - Delays:  $\max(M, N)$
- Direct forms I and II are equivalent in terms of input and output and differ only in their internal structure.
- These forms in Matlab are implemented with the function **filter()**

# Direct Form II



Stage-diagram of IIR system in Direct form II

# Example 1

An LSI system is described by the transfer function:

$$H(z) = \frac{1 + 0.9z^{-1}}{(1 + 0.1z^{-1} + 0.5z^{-2})(1 - 0.6z^{-1})}$$

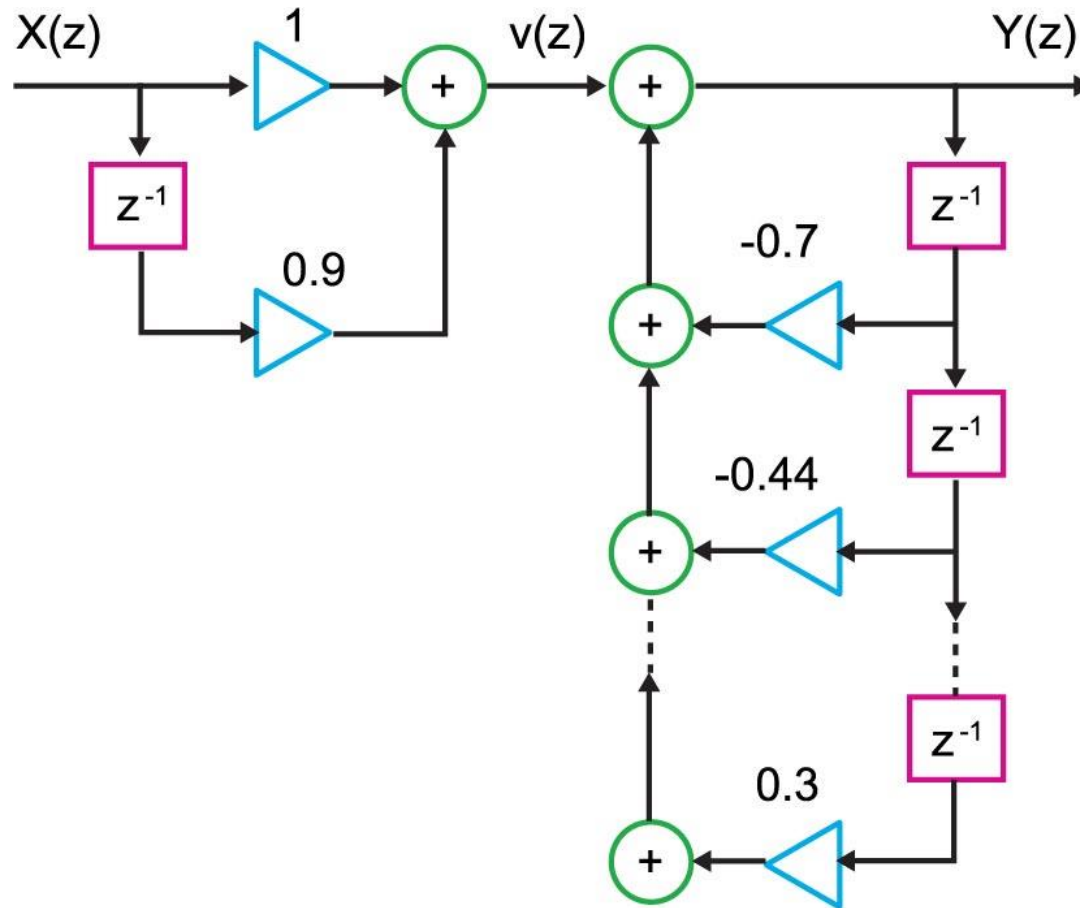
- (a) Draw the step diagrams of Direct form I and II.
- (b) For each form calculate the number of multiplications and additions required to calculate each output sample, as well as the number of delay registers.

**Answer:** (a) We do the operations on the denominator, so the transfer function is written:

$$H(z) = \frac{1 + 0.9z^{-1}}{1 + 0.7z^{-1} + 0.44z^{-2} - 0.3z^{-3}}$$

The step diagrams of Direct Form I and II are shown in the next figure.

# Example 1 (continued)



(a) Stage-diagram of Direct form I



# Example 1 (continued)

(b) According to stage-diagrams (a) and (b), the number of computations in Direct form I is:

- Multiplications: 5 for each output sample
- Additions: 4 for each output sample
- Delays: 4

and in Direct form II is:

- Multiplications: 5 for each output sample
- Additions: 4 for each output sample
- Delays: 3

# Cascade form

If  $h[n]$  is a real sequence, then  $H(z)$  is resolved into a product of second-order factors with real coefficients. Specifically and for  $M = N(N\text{even})$ , we have:

$$H(z) = \frac{b_0 + b_1z^{-1} + \dots + b_Mz^{-N}}{1 + a_1z^{-1} + \dots + a_Nz^{-N}} = b_0 \prod_{k=1}^{N/2} \frac{1 + B[k, 1]z^{-1} + B[k, 2]z^{-2}}{1 + A[k, 1]z^{-1} + A[k, 2]z^{-2}}$$

where  $A[k, 1], A[k, 2], B[k, 1], B[k, 2]$  are real numbers representing the coefficients of the second-order factors. Each factor is written:

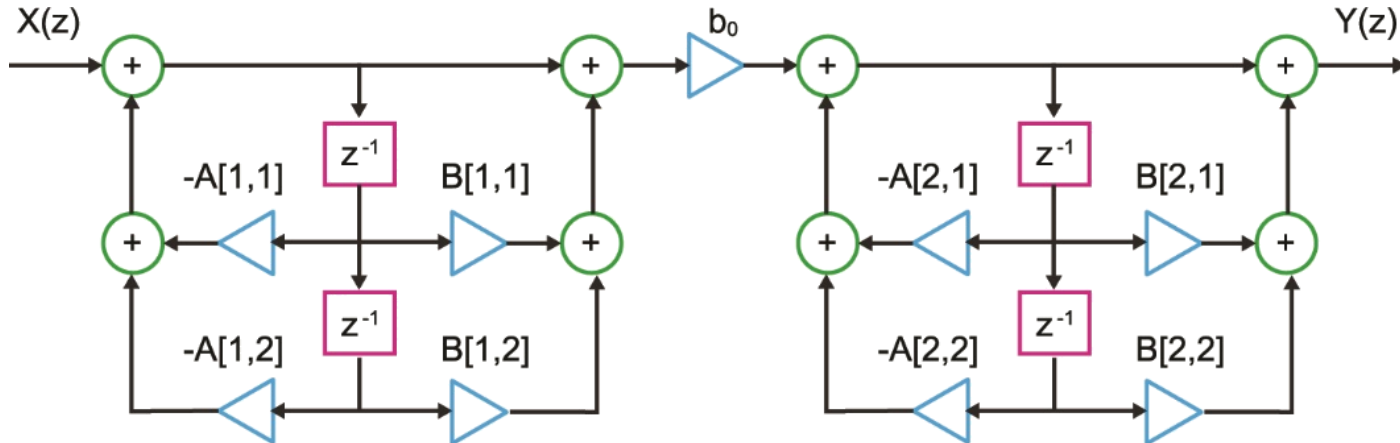
$$H_k(z) = \frac{Y_{k+1}(z)}{Y_k(z)} = \prod_{k=1}^{N/2} \frac{1 + B[k, 1]z^{-1} + B[k, 2]z^{-2}}{1 + A[k, 1]z^{-1} + A[k, 2]z^{-2}}, \quad k = 1, 2, \dots, N/2$$

Also, it applies:  $Y_1(z) = b[0] X(z)$  and  $Y_{(N/2)+1}(z) = Y(z)$

Each term is implemented in Direct Form II and  $H_k(z)$  the terms are connected in series to create the cascade diagram of the next form (for  $N = 4$ ), which is called Cascade Form.



# Cascade form



IIR structure of cascaded system

- The cascade form provides flexibility in how the system is implemented because there are different ways to combine the poles and zeros as well as the order in which the subsystems are connected.
- To calculate in Matlab the coefficients of the Cascade form we use the function `tf2sos()`.

# Parallel Form

It turns out that the transfer function  $H(z)$  can be written as a sum of second-order factors:

$$H(z) = \sum_{k=1}^{N/2} \frac{B[k, 0] + B[k, 1]z^{-1}}{1 + A[k, 1]z^{-1} + A[k, 2]z^{-2}} + \sum_0^{M-N} C_k z^{-k}$$

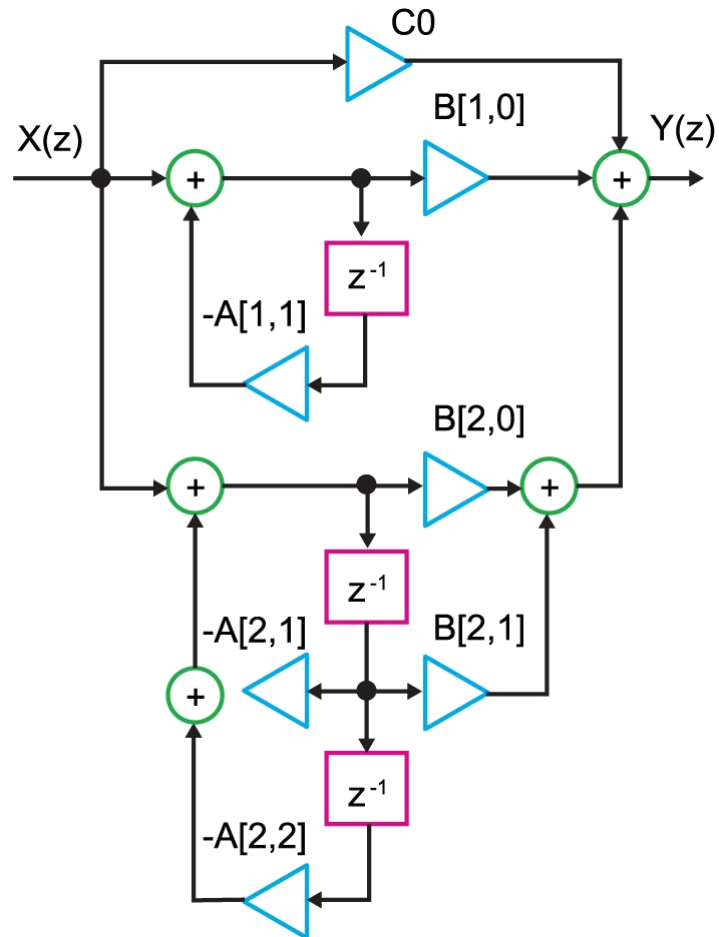
where  $A[k, 1], A[k, 2], B[k, 0], B[k, 1]$  are real numbers representing the coefficients of the second-order factors. Each second-order factor is written:

$$H_k(z) = \frac{Y_{k+1}(z)}{Y_k(z)} = \frac{B[k, 0] + B[k, 1]z^{-1}}{1 + A[k, 1]z^{-1} + A[k, 2]z^{-2}}, \quad k = 1, 2, \dots, N/2$$

Equally, it works  $Y_k(z) = H_k(z)X(z)$  and  $Y(z) = \sum Y_k(z)$ ,  $M < N$

Each term  $H_k(z)$  is realized in Direct form II and the terms are connected in series to create the ladder diagram of the following figure (for  $M = N = 4$ ), which is called **Parallel Form**.

# Parallel Form



IIR system structure in parallel form

# Finite Impulse Response (FIR) Filters

- Direct Form
- Cascade form
- Linear Phase Form
- Frequency Sampling Form

# Description of FIR Systems

Time-Domain:

$$y[n] = \sum_{m=0}^M b_m x[n - m] = \sum_{m=0}^M h[m] x[n - m] \quad (1)$$

Frequency-Domain:

$$H(z) = b_0 + b_1 z^{-1} + \dots + b_M z^{-M} = \sum_{m=0}^M b_m z^{-m} \quad (2)$$

- $M$  multiplications and  $M + 1$  additions are required for each output sample. The order of the FIR system is  $M - 1$  while its length is  $M$ .
- FIR systems are always stable (because they have no poles) and can be designed to have a linear phase response. Also, they are simpler than IIRs.

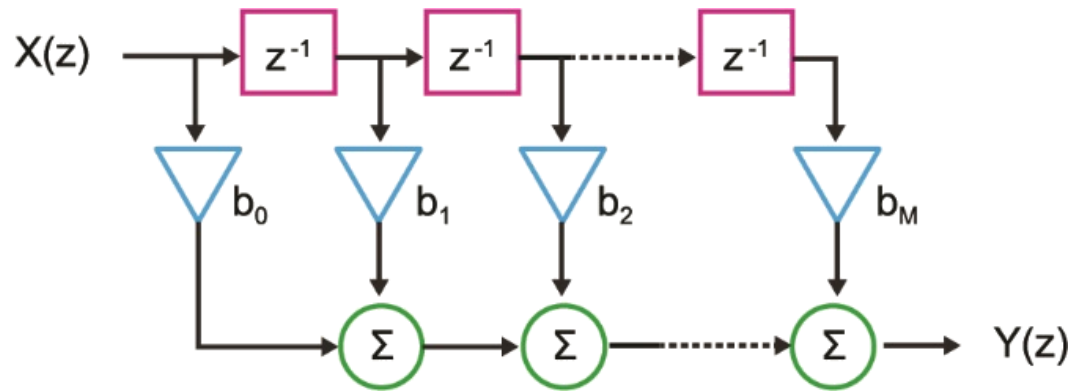
# FIR filter implementation structures

They are schematic diagrams of the different but equivalent ways the transfer function  $H(z)$  can be organized.

- **Direct Form:** Resulting from the direct application of the LDECC in a step chart.
- **Cascade form:** The transfer function is factorized into smaller second-order sections. Each part is implemented in a Direct form and the overall system results from the combination of the individual parts.
- **Linear phase:** When the impulse response satisfies some symmetry properties then the phase of the system is linear. Due to the symmetry, the number of required calculations is reduced.
- **Frequency Sampling:** Based on the DFT calculation  $H[k]$  of the impulse response.

# Direct Form

The Direct form is derived directly from the LDECC and is implemented as a branching delay line, as shown in the figure.



FIR system structure in Direct form

- The number of calculations in the Direct form is:
  - Multiplications:  $M + 1$  for each output sample
  - Additions:  $M$  for each output sample
  - Delays:  $M$
- If there are symmetries in the impulse response, then the number of actions can be reduced.

# Example 2

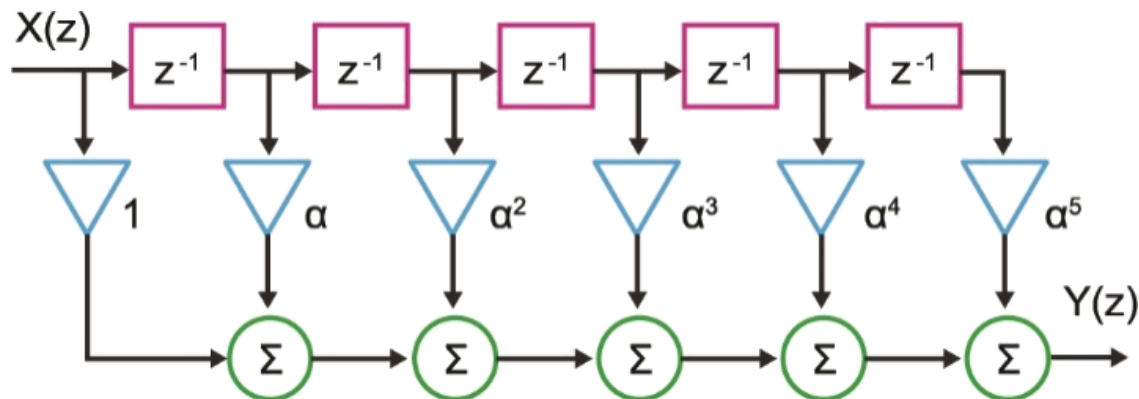
(a) Draw the Direct form of the FIR system with impulse response:

$$h[n] = \begin{cases} a^n, & 0 \leq n \leq 5 \\ 0, & \text{elsewhere} \end{cases}$$

**Answer:** The impulse response is written:

$$\begin{aligned} h[n] &= \alpha^n [u[n] - n[n - 6]] \\ &= \delta(0) + a\delta(1) + a^2\delta(2) + a^3\delta(3) + a^4\delta(4) + a^5\delta(5) \end{aligned}$$

from which the Direct form step diagram (N=6) is derived:





## Example 2 (continued)

(b) Calculate the number of multiplications and additions required to calculate each output sample as well as the number of delay registers.

**Answer:** From the step diagram it follows that the number of calculations in the Direct form is:

- Multiplications: 6 for each output sample
- Additions: 5 for each output sample
- Delays: 5

# Cascade Form

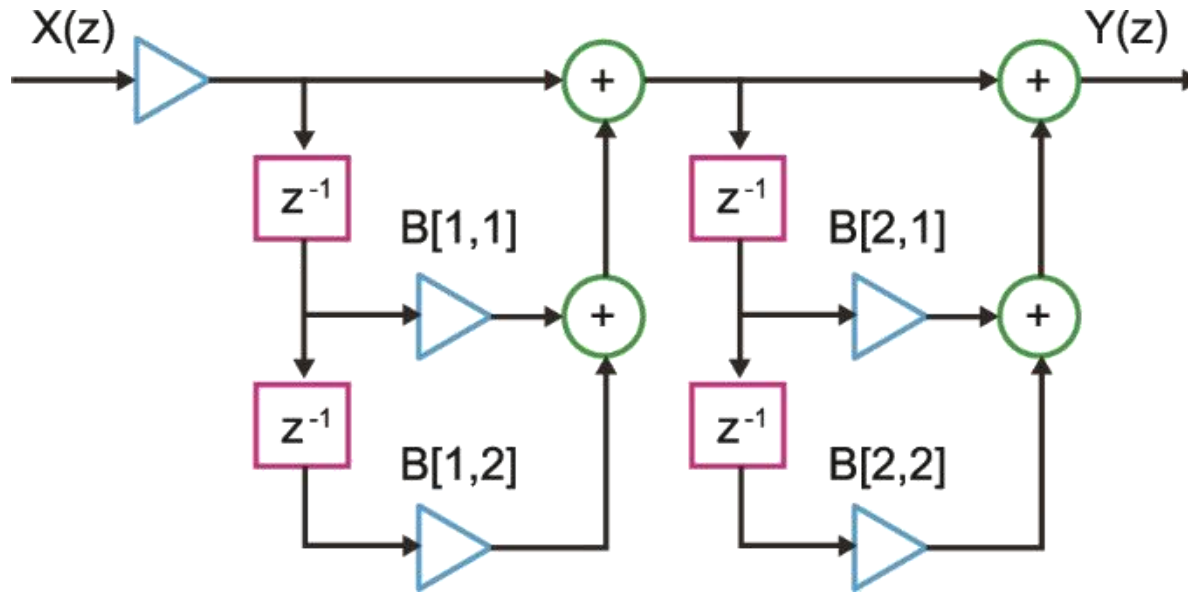
- If the impulse response  $h[n]$  is a real sequence then the  $H(z)$  can be written as a product of second-order factors with real coefficients. For  $N$  even, we have:

$$H(z) = b_0 \prod_{k=1}^{N/2} (1 + B[k, 1]z^{-1} + B[k, 2]z^{-2})$$

where  $B[k, 1], B[k, 2]$  are real numbers representing the coefficients of the second-order factors.

- Each term  $H_k(z)$  is implemented in Direct form II and the terms are connected in series to create the cascade diagram of the next figure (example for  $N = 4$ ), which is called **Cascade Form**.
- To calculate in Matlab the coefficients of the Cascade form we use the function `tf2sos()`.

# Cascade Form



FIR system structure in cascade form ( $N=4$ )

# Linear Phase Form

- If the impulse response  $h[n]$  satisfies symmetry properties then we can simplify the form of the stage-diagram.
- The impulse response can be either symmetric or antisymmetric according to the respective equations:

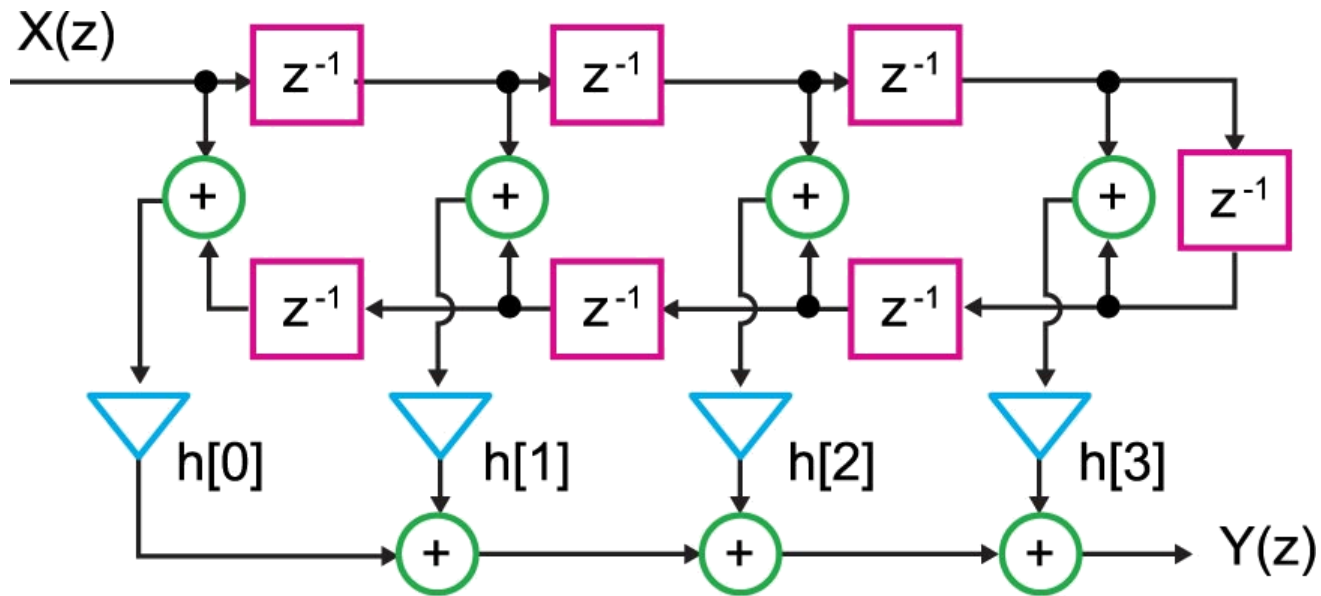
$$h[n] = h[N - n] \quad \text{and} \quad h[n] = -h[N - n]$$

- For symmetric impulse response and  $N$  even, the LDECC (1) is written:

$$y[n] = \sum_{m=0}^{(M/2)-1} h[m] \{x[n - m] + x[n - N + m]\} + h\left[\frac{N}{2}\right] x\left[n - \frac{N}{2}\right]$$

- The equation is implemented with the step diagram of the next figure and with 50% of the operations of the Direct form.
- If the impulse response is antisymmetric and  $N$  even, we have type II filters.
- Similarly, for odd  $N$ , we have filters of type III and IV, respectively.

# Linear Phase Form



Structure of FIR system in type I filter ( $N = 4$ )

# Frequency Sampling Form

- It is based on the fact that  $H(z)$  can be reconstructed from equidistant samples of it on the unit circle, which have been calculated by DFT  $M$ -points  $H[k]$  of the impulse response  $h[n]$ .
- It turns out that the equation holds (where:  $W_M^{-k} = e^{\frac{j2\pi}{N}k}$ ):

$$H(z) = \frac{1}{M} (1 - z^{-M}) \sum_{k=0}^{M-1} \frac{H[k]}{1 - W_M^{-k} z^{-1}}$$

- With this equation we describe an FIR filter in a recursive form. Actually the poles  $W_M^{-k}$  are canceled by the roots of the equation  $1 - z^{-M} = 0$ , so the system always remains stable.
- The equationship is implemented by the series connection of an FIR filter  $(1 - z^{-M})/M$  with a parallel network of one-pole filters:

$$H_k(z) = \frac{H[k]}{1 - W_M^{-k} z^{-1}}$$

# Frequency Sampling Form

- If the impulse response  $h[n]$  is real, then exploiting the symmetry properties of  $W_M^{-k}$ , it turns out that the transfer function is given by the equation:

$$H(z) = \frac{1}{M} (1 - z^{-M}) \left\{ \sum_{k=0}^L 2|H[k]| H_k(z) + \frac{H[0]}{1 - z^{-1}} + \frac{H[M/2]}{1 - z^{-1}} \right\}$$

where  $L = (M - 1)/2$  for  $M$  even and  $L = M/2 - 1$  for  $M$  odd.

- The second-order factors  $H_k(z)$  are given by the equation:

$$H_k(z) = \frac{\cos[\angle H[k]] - z^{-1} \cos \left[ \angle H[k] - \frac{2\pi k}{M} \right]}{1 - 2z^{-1} \cos \left( \frac{2\pi k}{M} \right) + z^{-2}}$$

# Lattice Filters

- Lattice FIR filter
- Lattice All Pole

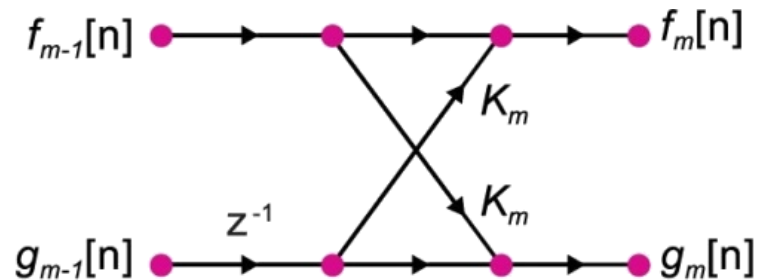


# Grid Filters

- The lattice filters are very popular in various applications because they have important properties, such as the smaller number of coefficients compared to IIR and FIR filters for the same performance, the possibility of modular wiring, stability and low sensitivity to the quantization of their coefficients.
- They are widely used in digital speech analysis and synthesis, as their low number of coefficients allows real-time implementation of complex processes required in speech processing.
- Separated in:
  - Type of FIR
  - All - pole
  - Poles and zeros IIR filters

# Lattice FIR

- An FIR mesh filter of length  $M$  or order  $M-1$  is created by a series connection of  $M-1$  elementary quadrupoles (stages).



Elementary quadrupole (stage) lattice

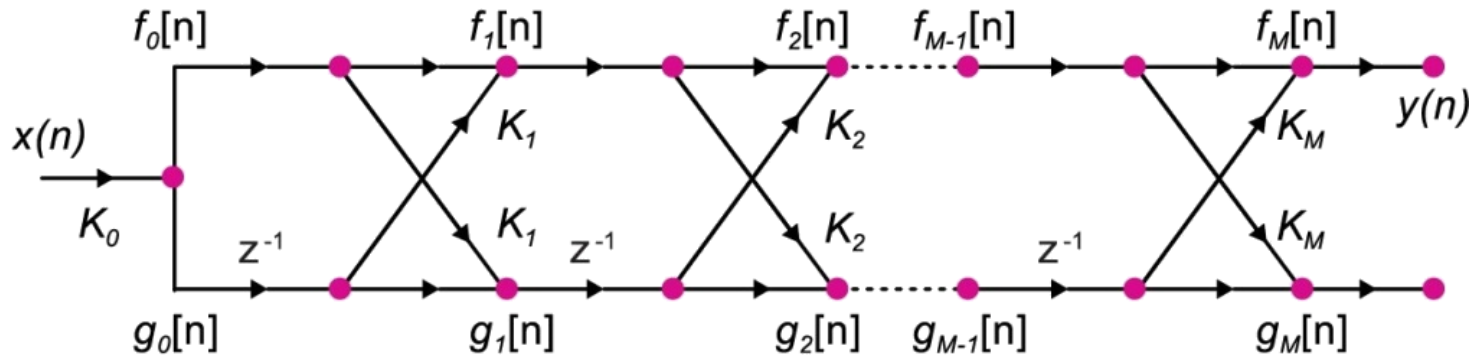
- The following difference equations apply:

$$f_m[n] = f_{m-1}[n] + K_m g_{m-1}[n-1], \quad m = 1, 2, \dots, M-1 \quad (1)$$

$$g_m[n] = g_{m-1}[n-1] + K_m f_{m-1}[n], \quad m = 1, 2, \dots, M-1 \quad (2)$$

# Lattice FIR

- The figure shows the complete connection in series of  $M$  stages and the creation of a mesh filter, of length  $M$ .



Grid filter of length  $M$

- The coefficients  $K_m$ ,  $m = 1, 2, \dots, M - 1$  are called **reflection coefficients** and define the grid filter. At the input of the first stage, the input signal  $x[n]$  multiplied by a scalar coefficient is applied  $K_0$ , that is:  $f_0[n] = g_0[n] = K_0x[n]$ .
- At the output of the last stage the output signal is produced:  $y[n] = f_M[n]$ .

# Lattice FIR

- If we define as  $A_m(z)$  the transfer function connecting the input  $x[n]$  to an intermediate output  $f_m[n]$ ,  $m = 1, 2, \dots, M - 1$ , from the equation:

$$F_m(z) = A_m(z) X(z)$$

and solve the difference equations (1) and (2) we obtain the following iterative formula for finding  $A_m(z)$ , which is called **positive step recursion** (step up down recursion):

$$A_m(z) = A_{m-1}(z) + K_m z^{-m} A_{m-1}(z^{-1}) \quad (3)$$

- The above recursive procedure is initialized with  $A_0(z) = 1$  and calculates the **total transfer function** between input and output of the mesh  $A_M(z)$  from the reflection coefficients  $K_m$ ,  $m = 1, 2, \dots, M - 1$ .
- The regression equation (3) as a function of the coefficients  $a_m[i]$  is:
  - $a_m[i] = a_{m-1}[i] + K_m a_{m-1}[m - i]$ ,  $i = 1, 2, \dots, m - 1$  (4)
  - $a_m[m] = K_m$  (5)

# Lattice FIR Filter

- In the case where the transfer function is known  $A_M(z)$  and the reflection coefficients are requested to be calculated  $K_m$ , we apply the **step down recursion** from the equation:

$$A_{m-1}(z) = \frac{1}{1 - K_m^2} \{A_m(z) - K_m z^{-m} A_m(z^{-1})\}, \quad m = M, \dots, 1 \quad (5)$$

- As in equations (3) and (4), the regression equation (5) expressed in terms of coefficients  $a_m[i]$  is:

- $\alpha_{m-1}[i] = \frac{1}{1 - K_m^2} \{a_m[i] - K_m a_m[m - i]\}, \quad i = 1, 2, \dots, m - 2 \quad (6)$

- $a_{m-1}[m - 1] = K_{m-1} \quad (7)$

# Lattice FIR Filter

- The mapping of the reflection coefficients of the lattice FIR filter to the transfer function coefficients of the Direct form of the FIR filter will be given. If the transfer function  $H(z)$  of the FIR filter in Direct form is:

$$H(z) = \sum_{m=0}^{M-1} b_m z^{-m}$$

and convert it into the following form, taking the coefficient as a common factor  $b_0$ :

$$H(z) = b_0 \left( 1 + \sum_{m=1}^{M-1} \frac{b_m}{b_0} z^{-m} \right) = b_0 \left( 1 + \sum_{m=1}^{M-1} \alpha_{M-1}[m] z^{-m} \right)$$

where:

$$\alpha_{M-1}[m] = \frac{b_m}{b_0}, \quad m = 1, 2, \dots, M - 1$$

# Lattice FIR Filter

- then the coefficients of the grid filter  $K_m$ ,  $m = 1, 2, \dots, M - 1$  in terms of the known coefficients of the Direct form  $b_m$ ,  $m = 1, 2, \dots, M - 1$  are given by the equations:

$$K_0 = b_0 \quad (8)$$

$$K_{M-1} = \alpha_{M-1}[M - 1] = \frac{b_{M-1}}{b_0} \quad (9)$$

$$K_m = \alpha_m[m], \quad m = M - 2, \dots, 1 \quad (10)$$

# Example 3

The reflection coefficients of a second-order FIR grating filter are  $K_1 = 1/4$  and  $K_2 = 1/8$ . Find the first-order  $A_1(z)$  and second-order transfer functions  $A_2(z)$ , that relate the input  $x[n]$  to  $f_1[n]$  and  $f_2[n]$ , respectively.

**Answer:** To calculate it,  $A_1(z)$  we put it  $m = 1$  in equation (3) and find:

$$A_1(z) = A_0(z) + K_1 z^{-1} A_0(z^{-1}) \quad (a)$$

The initial condition is  $A_0(z) = 1$ , therefore and  $A_0(z^{-1}) = 1$ . From equation (a) it is:

$$A_1(z) = 1 + \frac{1}{4} z^{-1} \quad (b)$$

From equation (b) we find that:

$$A_1(z^{-1}) = 1 + \frac{1}{4} z \quad (c)$$

Similarly, for it  $A_2(z)$  we put  $m = 2$  in equation (3) and we have:

$$A_2(z) = A_1(z) + K_2 z^{-2} A_1(z^{-1}) \quad (d)$$

We substitute equations (b) and (c) into equation (d) and find:

$$A_2(z) = \left(1 + \frac{1}{4} z^{-1}\right) + \frac{1}{8} z^{-2} \left(1 + \frac{1}{4} z\right) = 1 + \frac{9}{32} z^{-1} + \frac{1}{8} z^{-2}$$



# Example 4

To find the reflection coefficients of the second order FIR filter with transfer function:

$$A_2(z) = 1 - \frac{1}{2}z^{-2}$$

**Answer:** We put  $m = 2$  in equation (3) and we have:

$$A_2(z) = A_1(z) + K_2 z^{-2} A_1(z^{-1}) \quad (\alpha)$$

It  $A_1(z)$  is found by putting  $m = 1$  in equation (3):

$$A_1(z) = A_0(z) + K_1 z^{-1} A_0(z^{-1}) \quad (b)$$

Since  $A_0(z) = 1$  and  $A_0(z^{-1}) = 1$ , equation (a) is written:

$$A_1(z) = 1 + K_1 z^{-1} \quad (c)$$

## Example 4 (continued)

From equation (b) we find that:

$$A_1(z^{-1}) = 1 + K_1 z \quad (d)$$

We substitute equations (b), (c) and (d) into equation (a) and find:

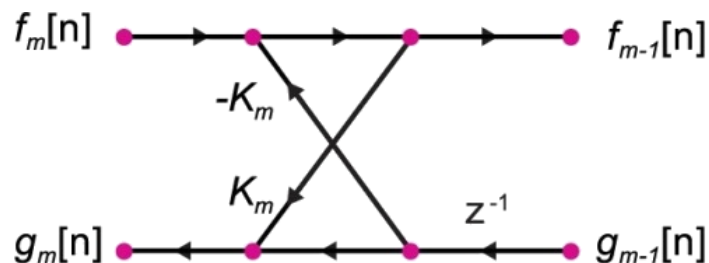
$$A_2(z) = 1 + K_1 z^{-1} + K_2 z^{-2}(1 + K_1 z) = 1 + (K_1 + K_1 K_2)z^{-1} + K_2 z^{-2} \quad (e)$$

We equate the corresponding coefficients of the given transfer function  $A_2(z)$  and equation (e) and find:

$$K_2 = -\frac{1}{2}, \quad K_1 = 0$$

# Lattice All Pole Filter

- A mesh filter of only  $N$  poles is created by a series connection of  $N-1$  elementary quadrupoles.



Elementary lattice quadrupole

- The following recursive equations apply:
  - $f_N[n] = x[n]$
  - $f_{m-1}[n] = f_m[n] - K_m g_{m-1}[n-1], m = N, N-1, \dots, 1$
  - $g_m[n] = K_m f_{m-1}[n] - g_{m-1}[n-1], m = N, N-1, \dots, 1$
  - $y[n] = f_0[n] = g_0[n]$

where the parameters  $K_m, m = 1, 2, \dots, M-1$  are the reflection coefficients of the pole-only grating filter.

# Lattice All Pole Filter

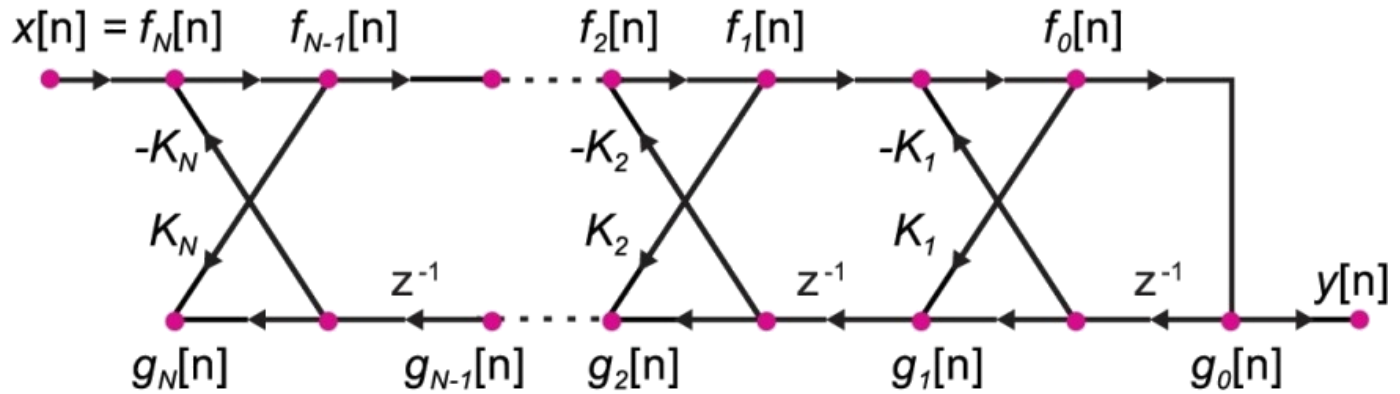
- These coefficients are calculated from the recursive equations (7) to (9), except for the coefficient  $K_0$ , which is equal to 1 ( $K_0 = 1$ ).
- As the quadrupole has no zeros, its frequency response is given by:

$$H(z) = \frac{1}{A_N(z)} = \frac{1}{1 + \sum_{m=1}^N \alpha_N[m] z^{-m}}$$

where the transfer function  $A_N(z)$  is calculated by the algorithm described in equation (3).

- It is clear that the all-pole lattice filter is the inverse of the FIR lattice filter, except for the coefficient  $b_0$ .
- Therefore, a grid structure of a all-pole IIR filter can be produced by inverting a grid structure of an FIR filter.

# Lattice All Pole Filter



N-order all-pole filter of N order

# Lattice IIR Filter

- The transfer function of an IIR filter with  $N$  poles and  $M$  zeros (generally we assume that  $N \leq M$ ) is given by the equation:

$$H(z) = \frac{B_M(z)}{A_N(z)} = \frac{\sum_{m=0}^M b_m z^{-m}}{1 + \sum_{m=1}^N \alpha_N[m] z^{-m}}$$

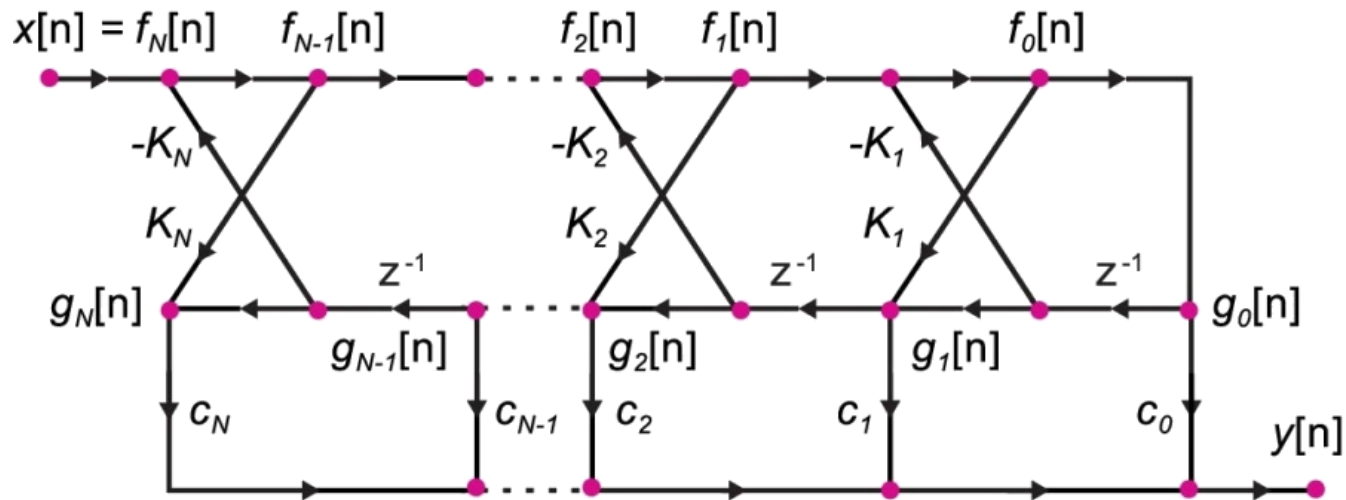
- From this equation it follows that an IIR lattice filter consists of two parts:
  - The first part corresponds to the term  $(1/A_N(z))$  and is a all-pole grid filter with reflection coefficients  $K_m$ ,  $1 \leq m \leq N$ .
  - The second part is a branched delay line (tapped delay line) with coefficients  $C_m$ , which produces at the grid output a weighted linear combination of the samples  $g_m[n]$  according to the equation:

$$y[n] = \sum_{m=0}^M C_m g_m[n] \quad (11)$$

# Lattice IIR Filter

- The coefficients  $C_m$  of the delay line identify the zeros of the transfer function  $H(z)$  and are shown to be calculated from the coefficients  $b_m$  of the Direct form by the equation:

$$C_m = b_m + \sum_{i=m+1}^M C_i a_i [i - m], \quad m = M - 1, \dots, 1, 0$$



Lattice IIR filter

# Example 5

Convert the following IIR pole-zero filter to lattice form:

$$H(z) = \frac{0.25 + 0.5z^{-1} - 0.4z^{-2}}{1 - 0.1z^{-1} + z^{-2}}$$

**Answer:** First we will convert the denominator coefficients to reflection coefficients. We apply the methodology presented in Example 4 to the function:

$$A_2(z) = 1 - 0.1z^{-1} + z^{-2}$$

also  $m = 2$  put  $m = 2$  in equation (3) and correspondingly we get:

$$A_2(z) = A_1(z) + K_2 z^{-2} A_1(z^{-1}) \quad (a)$$

$$A_1(z) = A_0(z) + K_1 z^{-1} A_0(z^{-1}) \quad (b)$$



## Example 5 (continued)

Because  $A_0(z) = 1$  and  $A_0(z^{-1}) = 1$ , we find:

$$A_1(z) = 1 + K_1 z^{-1} \quad (c)$$

$$A_1(z^{-1}) = 1 + K_1 z \quad (d)$$

We substitute equations (b), (c) and (d) into equation (a) and find:

$$A_2(z) = 1 + K_1 z^{-1} + K_2 z^{-2}(1 + K_1 z) = 1 + (K_1 + K_1 K_2)z^{-1} + K_2 z^{-2} \quad (e)$$

We equate the corresponding coefficients of the original function  $A_2(z)$  and equation (e) and find:

$$K_2 = 1, \quad K_1 = -0.05$$

## Example 5 (continued)

The coefficients  $a_m$  and  $b_m$  given in the pronunciation are:

$$a_m = \{\alpha_0, \alpha_1, \alpha_2\} = \{1, -0.1, 1\}$$

$$b_m = \{b_0, b_1, b_2\} = \{0.25, 0.5, -0.4\}$$

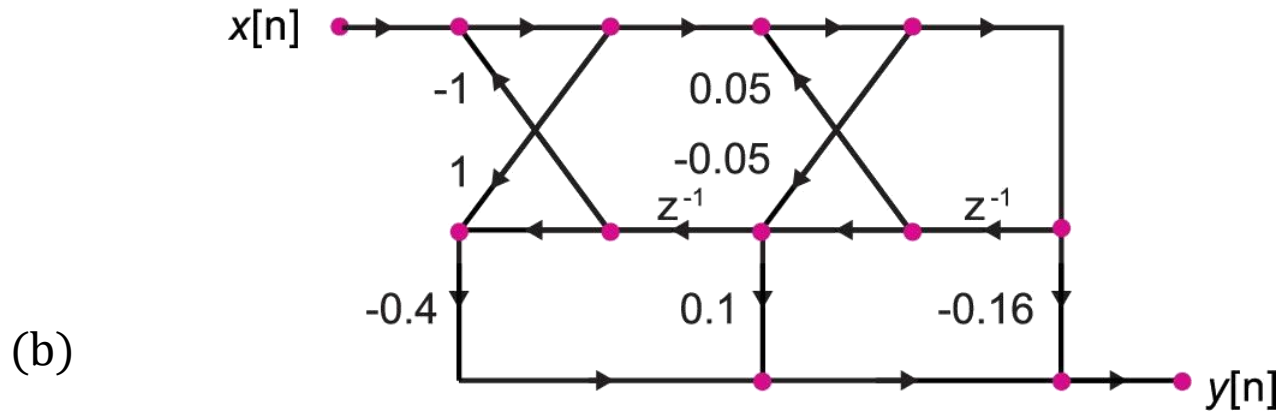
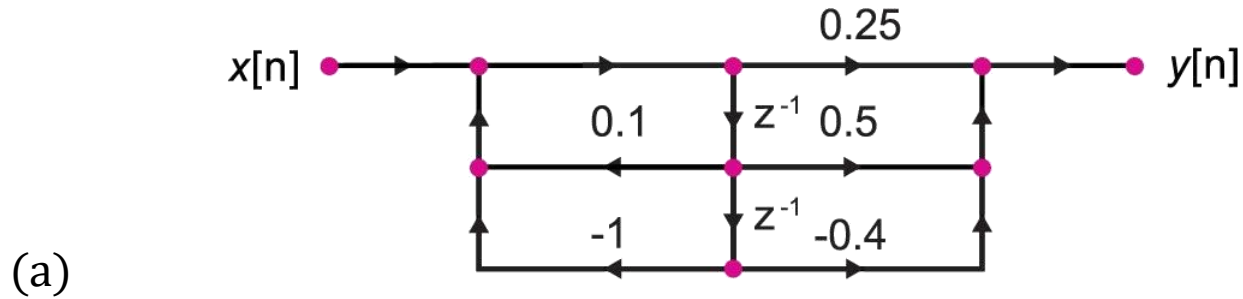
Finally, the coefficients  $C_2$  are calculated from the regression equation (11) for  $m = 2, 1, 0$  and are:

- $m = 2$ :  $C_2 = b_2 = -0.4$
- $m = 1$ :  $C_1 = b_1 + C_2 a_2[1] = 0.5 + (-0.4) 1 = 0.1$
- $m = 0$ :  $C_0 = b_0 + C_1 a_1[1] + C_2 a_2[2] = 0.25 + 0.1 (-0.1) + (-0.4) 1 = -0.16$

Therefore:

$$C_0 = -0.16, C_1 = 0.1, C_2 = -0.4$$

# Example 5 (continued)



IIR filter: (a) Direct form, (b) Lattice form