University of the Peloponnese
Electrical and Computer
Engineering Department

DIGITAL SIGNAL PROCESSING

Solved Examples

Teacher: M. Paraskevas

SET #9 - Discrete Fourier Transform

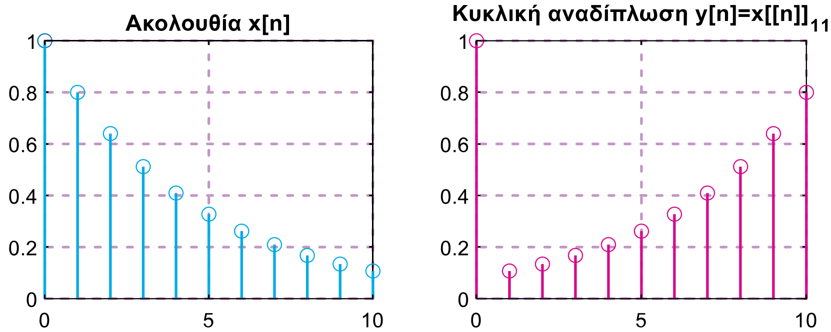
- DFT properties
- Relation of circular to linear convolution

1. DFT properties

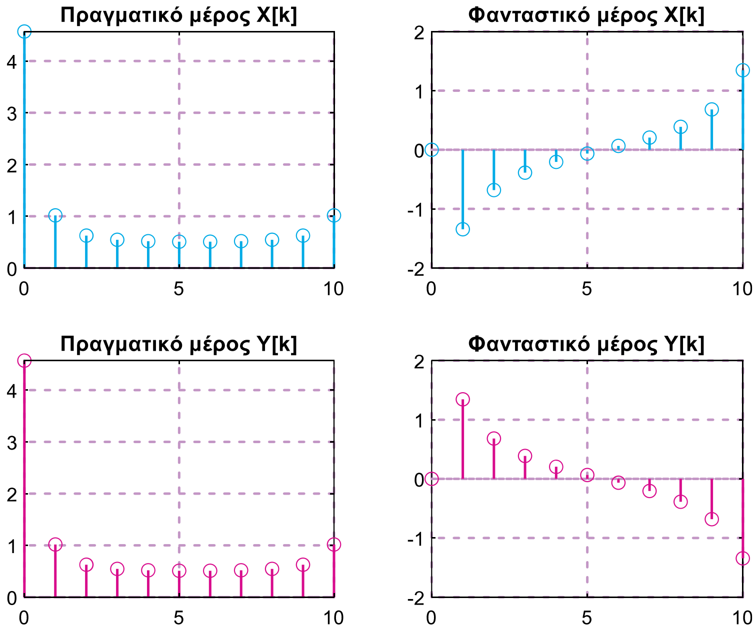
📖Example 1

Using the sequence$x[n] = (0.8)^n$ to $0 \leq n \leq 10$, confirm the circular folding property.

Answer :



Following$x[n]$ και κυκλική αναδίπλωση $y[n] = x[[-n]]_{11}$



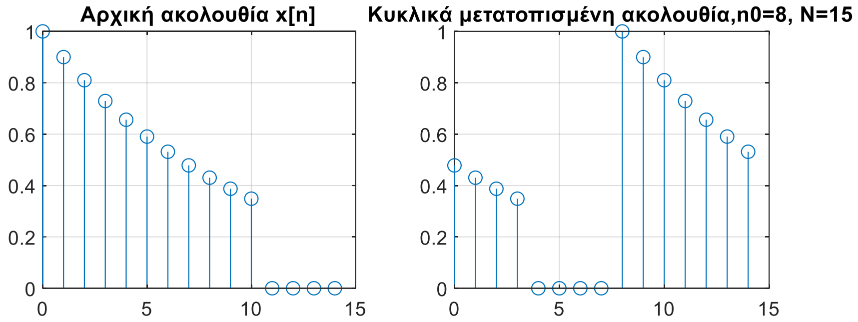
Real and imaginary part of DFTs $X[k]$ και $Y[k]$

Comparing real and imaginary part diagrams of DFTs $X[k]$and $Y[k]$, we find that the relation holds $Y[k] = X[N-k]$, so the circular folding property is confirmed.

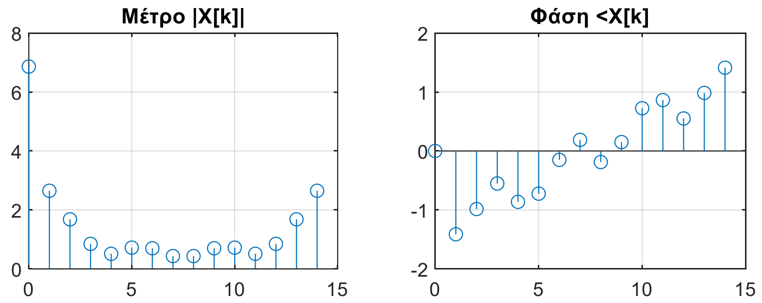
Example 2

Calculate the DFT of the circularly shifted sequence $y[n] = x[[n-8]]_{15}$,where$x[n] = (0.9)^n$ for $0 \leq n \leq 10$.

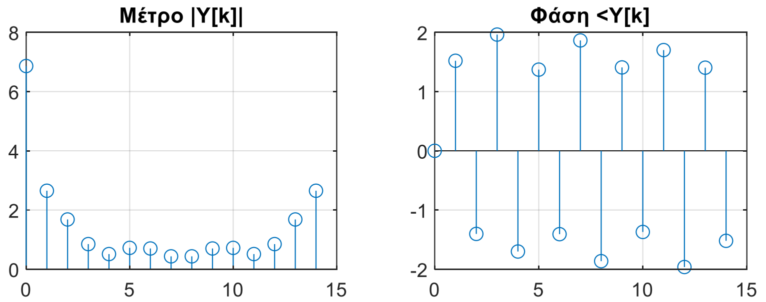
Answer :



Following$x[n]$ και κυκλικά μετατοπισμένη ακολουθία $y[n] = x[[n-8]]_{15}$



Measure and phase of the DFT $X[k]$



Measure and phase of the DFT $Y[k]$

DFT gauge plots $X[k]$and $Y[k]$we find that they are the same, while the phase diagrams show a phase shift equal to the phase of the term W_{15}^{8k}.

2. Relation of circular to linear convolution

Example 3

Calculate the circular convolution of 4 points between the sequences $x[n] = \{\hat{0}, 1, 2, 3\}$and $h[n] = \{1, \hat{2}, 0, -1\}$using the DFT .

Answer : (a) We rewrite the given sequences as:

$$x[n] = \{0, 1, 2, 3\}, n = 0, 1, 2, 3$$

$$h[n] = \{1, 2, 0, -1\}, n = -1, 0, 1, 2$$

We note that the sequence $h[n]$can be thought of as the circular shift by one unit to the left of a sequence $g[n] = \{1, 2, 0, -1\}, n = 0, 1, 2, 3$, that is, it is:

$$h[n] = g\big[[n + 1]\big]_4$$

We calculate through the DFT the output from the circular convolution $y[n] = x[n]④$ $g[n] \overset{DFT}{\longleftrightarrow} X[k]\, G[k]$and then apply a circular shift to the left by one unit.

```
% Length of circular convolution
N = 4;
% Set time scale and sequences x [ n ] and g [ n ]
n = [0, 1, 2, 3]? x = [0, 1, 2, 3]? g = [1, 2, 0, -1];
DFT calculation of N points X [ k ] and G [ k ]
X = fft( x, N ); G = fft(g, N);
% Multiply Y[ k ] = X [ k ]. G [ k ]
Y = X . * G ;
% Inverse DFT calculation
y = ifft ( Y , N );
% Circular shift by -1
y = circleshift (y,-1)
```

Result : `y = -1 1 7 5`

We notice that the result is in agreement with the calculation result of circular convolution in the time domain.