UNIVERSITY OF THE PELOPONNESE – SCHOOL OF ENGINEERING
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

SIGNALS AND SYSTEMS
Solved Examples – Tutorial Sheet 12

Instructor: Michalis Paraskevas, Professor

ANALYSIS OF LTI SYSTEMS USING THE FOURIER TRANSFORM

- Frequency response
- Response of an LTI system to periodic and aperiodic inputs
- Describing a linear differential equation using the Fourier transform
- Inverse systems

1. Frequency Response

📖 Example 1

Calculate the output of an LTI system with impulse response $h(t) = 2e^{-3t}u(t)$ when its input is $x(t) = 2cos(3t + \pi/3)$.

Answer: From Table 4.3, the Fourier transform of the impulse response, which is the system frequency response, is:

$$H(\Omega) = \frac{2}{3 + j\Omega}$$

Its amplitude (magnitude) is:

$$|H(\Omega)| = \left|\frac{2}{3 + j\Omega}\right| = \frac{2}{|3 + j\Omega|} = \frac{2}{\sqrt{9 + \Omega^2}}$$

and its phase is:

$$\theta_H(\Omega) = \measuredangle H(\Omega) = -\tan^{-1}\left(\frac{\Omega}{3}\right)$$

The signal $x(t) = 2cos(3t + \pi/3)$ has frequency $\Omega_0 = 3$ (rad/s). Its exponential Fourier series expansion is:

$$x(t) = 2cos\left(3t + \frac{\pi}{3}\right) = e^{j\left(3t+\frac{\pi}{3}\right)} + e^{-j\left(3t+\frac{\pi}{3}\right)} = e^{j3t}e^{j\pi/3} + e^{-j3t}e^{-j\pi/3}$$

Using equation (5.72), the system output is:

$$y(t) = H(3)\left\{e^{j3t}e^{\frac{j\pi}{3}}\right\} + H(-3)\left\{e^{-j3t}e^{-\frac{j\pi}{3}}\right\}$$

We calculate:

$$H(3) = \frac{2}{3 + 3j} = \frac{\sqrt{2}}{3}e^{-\frac{j\pi}{4}} \qquad \text{and} \qquad H(-3) = \frac{2}{3 - 3j} = \frac{\sqrt{2}}{3}e^{\frac{j\pi}{4}}$$

Grouping the exponential terms gives:

$$y(t) = \frac{\sqrt{2}}{3} e^{-\frac{j\pi}{4}} \left\{ e^{j3t} e^{\frac{j\pi}{3}} \right\} + \frac{\sqrt{2}}{3} e^{\frac{j\pi}{4}} \left\{ e^{-j3t} e^{-\frac{j\pi}{3}} \right\}$$
$$= \frac{\sqrt{2}}{3} \{ e^{j(3t+\pi/12)} \} + \frac{\sqrt{2}}{3} \{ e^{-j(3t+\pi/12)} \}$$
$$= \frac{2\sqrt{2}}{3} \cos\left(3t + \frac{\pi}{12}\right)$$

The output frequency is the same as the input frequency.

The output amplitude equals the input amplitude multiplied by the magnitude of the frequency response at $\Omega_0 = 3$:

$$2 \cdot | H(3) | = 2 \cdot \frac{\sqrt{2}}{3} = \frac{2\sqrt{2}}{3}$$

and the output phase equals the input phase shifted by the phase of the frequency response at $\Omega_0 = 3$:

$$\frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

Comments:

- The output retains the input frequency. Since the system is LTI and its input is a single-frequency sinusoid, the output remains sinusoidal at the same frequency, $\Omega_0 = 3$ rad/s.
- The system changes only amplitude and phase. Its effect on the input is fully described by the frequency response at Ω_0. Here, the amplitude is scaled by $|H(3)| = \sqrt{2}/3$, and the phase is shifted by $\Theta_H(3) = -\pi/4$. Thus, the output is obtained by applying these two changes directly to the original sinusoid.
- The final output confirms eigenfunction theory. The solution gives:

  $$y(t) = \frac{2\sqrt{2}}{3} \cos\left(3t + \frac{\pi}{12}\right)$$

  showing that a sinusoidal input to an LTI system produces an output with the same frequency but a new amplitude and phase.
- The frequency-response method is much shorter than time-domain convolution, especially for a purely sinusoidal input.

📖 Example 2

Calculate the impulse response of the system described by the differential equation:

$$\frac{d^2y(t)}{dt^2} + 8\frac{dy(t)}{dt} + 15y(t) = 15\,x(t) - 2\frac{dx(t)}{dt}$$

Answer: (a) We solved this problem in the time domain and obtained:

$$h(t) = \frac{1}{2}(21e^{-3t} - 25e^{-5t})\,u(t)$$

Here, we solve the same problem in the frequency domain using the Fourier transform, to compare the solutions.

Applying the Fourier transform to both sides of the differential equation and using its linearity and differentiation properties gives:

$$F\left\{\frac{d^2y(t)}{dt^2}+8\frac{dy(t)}{dt}+15y(t)\right\}=F\left\{15x(t)-2\frac{dx(t)}{dt}\right\}\Rightarrow$$
$$F\left\{\frac{d^2y(t)}{dt^2}\right\}+8F\left\{\frac{dy(t)}{dt}\right\}+15F\{y(t)\}=15F\{x(t)\}-2F\left\{\frac{dx(t)}{dt}\right\}$$

Using the differentiation property of the Fourier transform:

$$F\left\{\frac{d^nx(t)}{dt^n}\right\}\overset{F}{\longleftrightarrow}(j\Omega)^nX(\Omega)$$

we obtain:

$$(j\Omega)^2\,Y(\Omega)+8j\Omega\,Y(\Omega)+15\,Y(\Omega)=15\,X(\Omega)-2j\Omega\,X(\Omega)\Rightarrow$$
$$Y(\Omega)(-\Omega^2+8j\Omega+15)=X(\Omega)(15-2j\Omega)\Rightarrow$$
$$H(\Omega)=\frac{Y(\Omega)}{X(\Omega)}=\frac{15-2j\Omega}{-\Omega^2+8j\Omega+15}=\frac{15-2j\Omega}{(3+j\Omega)(5+j\Omega)}$$

To calculate the impulse response $h(t)$, we apply the inverse Fourier transform to the frequency response $H(\Omega)$. We first decompose the frequency response into partial fractions:

$$H(\Omega)=\frac{C_1}{3+j\Omega}+\frac{C_2}{5+j\Omega}$$

We calculate C_1 and C_2:

$$C_1=\frac{15-2j\Omega}{(3+j\Omega)\,(5+j\Omega)}(3+j\Omega)\Big|_{j\Omega=-3}=\frac{15-2(-3)}{5-3}=\frac{21}{2}$$
$$C_2=\frac{15-2j\Omega}{(3+j\Omega)\,(5+j\Omega)}(5+j\Omega)\Big|_{j\Omega=-5}=\frac{15-2(-5)}{3-5}=-\frac{25}{2}$$

Thus:

$$H(\Omega)=\frac{21}{2}\frac{1}{3+j\Omega}-\frac{25}{2}\frac{1}{5+j\Omega}$$

Therefore, using the inverse Fourier transform and Table 4.3, the impulse response is:

$$h(t)=\frac{1}{2}(21e^{-3t}-25e^{-5t})\,u(t)$$

This result agrees with the time-domain solution, confirming the equivalence of the two approaches and demonstrating the effectiveness of the Fourier method for LTI system analysis.

Comments:

- This example shows that the Fourier transform converts a linear differential equation with constant coefficients into an algebraic relation between $X(\Omega)$ and $Y(\Omega)$, greatly simplifying frequency-domain analysis.
- Calculating the frequency response $H(\Omega)$ and decomposing it into partial fractions is the key step, since it allows direct application of the inverse Fourier transform. This gives the impulse response, which fully describes the LTI system in the time domain.

📖 Example 3 (*)

An LTI system has frequency response:

$$H(\Omega) = \frac{5 + j\Omega}{6 + 5j\Omega - \Omega^2}$$

(a) Find the differential equation describing the system.
(b) Find the system impulse response $h(t)$.
(c) Find the system output for the input $x(t) = e^{-5t}u(t)$.

Answer: (a) We know that the frequency response is given by:

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)}$$

For the given frequency response, we have:

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{5 + j\Omega}{6 + 5j\Omega - \Omega^2} \Rightarrow$$
$$Y(\Omega)(6 + 5j\Omega - \Omega^2) = X(\Omega)(5 + j\Omega) \Rightarrow$$
$$(j\Omega)^2 Y(\Omega) + 5j\Omega Y(\Omega) + 6Y(\Omega) = j\Omega X(\Omega) + 5X(\Omega)$$

We convert this algebraic relation into a differential equation using the differentiation property of the Fourier transform:

$$F\left\{\frac{d^2y(t)}{dt^2}\right\} = (j\Omega)^2 Y(\Omega), \quad F\left\{\frac{dy(t)}{dt}\right\} = (j\Omega)Y(\Omega), \quad F\left\{\frac{dx(t)}{dt}\right\} = (j\Omega)X(\Omega)$$

which gives:

$$F\left\{\frac{d^2y(t)}{dt^2}\right\} + 5F\left\{\frac{dy(t)}{dt}\right\} + 6y(t) = F\left\{\frac{dx(t)}{dt}\right\} + 5\,x(t)$$

Therefore, the differential equation describing the system is:

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 5\,x(t)$$

(b) We calculate the impulse response $h(t)$ by taking the inverse Fourier transform of the given frequency response $H(\Omega)$. Decomposing it into partial fractions gives:

$$H(\Omega) = \frac{5 + j\Omega}{6 + 5j\Omega - \Omega^2} = \frac{5 + j\Omega}{(2 + j\Omega)(3 + j\Omega)} = \frac{3}{2 + j\Omega} - \frac{2}{3 + j\Omega}$$

We obtain:

$$\begin{aligned} h(t) &= F^{-1}\{H(\Omega)\} \\ &= 3\,F^{-1}\left\{\frac{1}{2 + j\Omega}\right\} - 2\,F^{-1}\left\{\frac{1}{3 + j\Omega}\right\} \\ &= (3e^{-2t} - 2e^{-3t})u(t) \end{aligned}$$

(c) To calculate the system output $y(t)$, we work in the frequency domain because it simplifies the calculation. We first calculate the Fourier transform of the input, $X(\omega)$:

$$X(\Omega) = F\{e^{-5t}u(t)\} = \frac{1}{5 + j\Omega}$$

We then calculate the Fourier transform of the output:

$$Y(\Omega) = H(\Omega)\,X(\Omega) = \frac{5 + j\Omega}{(2 + j\Omega)(3 + j\Omega)}\,\frac{1}{(5 + j\Omega)} = \frac{1}{(2 + j\Omega)(3 + j\Omega)}$$

We write $Y(\Omega)$ as a sum of partial fractions:

$$Y(\Omega) = \frac{1}{(2 + j\Omega)(3 + j\Omega)} = \frac{1}{(2 + j\Omega)} - \frac{1}{(3 + j\Omega)}$$

Finally, the inverse Fourier transform gives the output $y(t)$:

$$y(t) = F^{-1}\{Y(\Omega)\} = \cdots = (e^{-2t} - e^{-3t})\,u(t)$$

Comments:

- When the frequency response $H(\Omega)$ is known, we can directly recover the system differential equation by moving from the frequency domain to the time domain using the Fourier transform's differentiation property.
- Partial-fraction decomposition of $H(\Omega)$ makes the impulse response $h(t)$ easy to calculate. The result $h(t) = (3e^{-2t} - 2e^{-3t})\,u(t)$ shows that the system's time-domain behavior is determined by exponential terms directly related to the poles of its frequency response.
- Working in the frequency domain greatly simplifies output calculations, since $Y(\Omega) = H(\Omega)X(\Omega)$ turns convolution into a simple product. This is especially useful when signals or systems have complex time-domain forms.

2. Response of an LTI System to Periodic and Aperiodic Inputs

📖 Example 4

Calculate the output of an LTI system with impulse response $h(t) = 2e^{-3t}u(t)$ when its input is $x(t) = 2cos(3t + \pi/3)$.

Answer: From Table 4.3, the Fourier transform of the impulse response, which is the system frequency response, is:

$$H(\Omega) = \frac{2}{3 + j\Omega}$$

Its amplitude (magnitude) is:

$$|H(\Omega)| = \left|\frac{2}{3 + j\Omega}\right| = \frac{2}{|3 + j\Omega|} = \frac{2}{\sqrt{9 + \Omega^2}}$$

and its phase is:

$$\theta_H(\Omega) = \measuredangle H(\Omega) = -\tan^{-1}\left(\frac{\Omega}{3}\right)$$

The signal $x(t) = 2cos(3t + \pi/3)$ has frequency $\Omega_0 = 3$ (rad/s). Its exponential Fourier series expansion is:

$$x(t) = 2cos\left(3t + \frac{\pi}{3}\right) = e^{j\left(3t+\frac{\pi}{3}\right)} + e^{-j\left(3t+\frac{\pi}{3}\right)} = e^{j3t}e^{j\pi/3} + e^{-j3t}e^{-j\pi/3}$$

Using equation (5.72), the system output is:

$$y(t) = H(3)\left\{e^{j3t}e^{\frac{j\pi}{3}}\right\} + H(-3)\left\{e^{-j3t}e^{-\frac{j\pi}{3}}\right\}$$

We calculate:

$$H(3) = \frac{2}{3 + 3j} = \frac{\sqrt{2}}{3}e^{-\frac{j\pi}{4}} \qquad \text{and} \qquad H(-3) = \frac{2}{3 - 3j} = \frac{\sqrt{2}}{3}e^{\frac{j\pi}{4}}$$

Grouping the exponential terms gives:

$$\begin{aligned} y(t) &= \frac{\sqrt{2}}{3}e^{-\frac{j\pi}{4}}\left\{e^{j3t}e^{\frac{j\pi}{3}}\right\} + \frac{\sqrt{2}}{3}e^{\frac{j\pi}{4}}\left\{e^{-j3t}e^{-\frac{j\pi}{3}}\right\} \\ &= \frac{\sqrt{2}}{3}\{e^{j(3t+\pi/12)}\} + \frac{\sqrt{2}}{3}\{e^{-j(3t+\pi/12)}\} \\ &= \frac{2\sqrt{2}}{3}\cos\left(3t + \frac{\pi}{12}\right) \end{aligned}$$

The output frequency is the same as the input frequency.

The output amplitude equals the input amplitude multiplied by the magnitude of the frequency response at $\Omega_0 = 3$:

$$2 \cdot |\, H(3)\,| = 2 \cdot \frac{\sqrt{2}}{3} = \frac{2\sqrt{2}}{3}$$

and the output phase equals the input phase shifted by the phase of the frequency response at $\Omega_0 = 3$:

$$\frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

Comments:

- The output retains the input frequency. Since the system is LTI and its input is a single-frequency sinusoid, the output remains sinusoidal at the same frequency, $\Omega_0 = 3$ rad/s.
- The system changes only amplitude and phase. Its effect on the input is fully described by the frequency response at Ω_0. Here, the amplitude is scaled by $|H(3)| = \sqrt{2}/3$, and the phase is shifted by $\Theta_H(3) = -\pi/4$. Thus, the output is obtained by applying these two changes directly to the original sinusoid.
- The final output confirms eigenfunction theory. The solution gives:

$$y(t) = \frac{2\sqrt{2}}{3}\cos\left(3t + \frac{\pi}{12}\right)$$

  showing that a sinusoidal input to an LTI system produces an output with the same frequency but a new amplitude and phase.
- The frequency-response method is much shorter than time-domain convolution, especially for a purely sinusoidal input.

3. Describing a Linear Differential Equation Using the Fourier Transform

📖 Example 5 (*)

Calculate the output of an LTI system with impulse response $h(t) = 2e^{-3t}u(t)$ when its input is $x(t) = (2e^{-t} + e^{-4t})\,u(t)$.

Answer: From Table 4.2, the system frequency response is:

$$H(\Omega) = \frac{2}{3 + j\Omega}$$

The Fourier transform of the input is:

$$X(\Omega) = \frac{2}{1 + j\Omega} + \frac{1}{4 + j\Omega}$$

and the Fourier transform of the output is:

$$Y(\Omega) = H(\Omega)X(\Omega) = \frac{2}{3 + j\Omega}\left(\frac{2}{1 + j\Omega} + \frac{1}{4 + j\Omega}\right)$$

$$= \cdots = \frac{6(3 + j\Omega)}{(1 + j\Omega)(3 + j\Omega)(4 + j\Omega)} = \frac{6}{(1 + j\Omega)(4 + j\Omega)}$$

$$= \frac{C_1}{1 + j\Omega} + \frac{C_2}{4 + j\Omega}$$

We calculate the coefficients C_1 and C_2 using:

$$C_1 = \frac{6}{(1 + j\Omega)(4 + j\Omega)}(1 + j\Omega)\Big|_{j\Omega=-1} = 2$$

$$C_2 = \frac{6}{(1 + j\Omega)(4 + j\Omega)}(4 + j\Omega)\Big|_{j\Omega=-4} = -2$$

Therefore, the Fourier transform of the output is:

$$Y(\Omega) = \frac{2}{1 + j\Omega} - \frac{2}{4 + j\Omega}$$

Thus, the output signal $y(t)$ is:

$$y(t) = (2e^{-t} - 2e^{-4t})\, u(t)$$

Comments:

- When the input is a sum of exponential signals, the output is easily calculated in the frequency domain using $Y(\Omega) = H(\Omega)X(\Omega)$, without directly calculating the time-domain convolution.
- The final result $y(t) = (2e^{-t} - 2e^{-4t})u(t)$ shows that the system changes the input components according to its frequency response, illustrating its usefulness for LTI system analysis.

📖 Example 6 (*)

Calculate the impulse response of the system described by the differential equation:

$$\frac{d^2y(t)}{dt^2} + 8\frac{dy(t)}{dt} + 15y(t) = 15\, x(t) - 2\frac{dx(t)}{dt}$$

Answer: (a) We have solved this problem in the time domain, obtaining:

$$h(t) = \frac{1}{2}(21e^{-3t} - 25e^{-5t})u(t)$$

We solve the problem in the frequency domain using the Fourier transform and compare the solutions. Applying the transform to both sides of the differential equation and using its linearity and differentiation properties gives:

$$F\left\{\frac{d^2y(t)}{dt^2} + 8\frac{dy(t)}{dt} + 15y(t)\right\} = F\left\{15x(t) - 2\frac{dx(t)}{dt}\right\} \Rightarrow$$

$$F\left\{\frac{d^2y(t)}{dt^2}\right\} + 8F\left\{\frac{dy(t)}{dt}\right\} + 15F\{y(t)\} = 15F\{x(t)\} - 2F\left\{\frac{dx(t)}{dt}\right\} \Rightarrow$$

$$(j\Omega)^2 Y(\Omega) + 8j\Omega\, Y(\Omega) + 15\, Y(\Omega) = 15\, X(\Omega) - 2j\Omega\, X(\Omega) \Rightarrow$$

$$Y(\Omega)(-\Omega^2 + 8j\Omega + 15) = X(\Omega)(15 - 2j\Omega) \Rightarrow$$

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{15 - 2j\Omega}{-\Omega^2 + 8j\Omega + 15} = \frac{15 - 2j\Omega}{(3 + j\Omega)(5 + j\Omega)}$$

To find the impulse response $h(t)$, we calculate the inverse Fourier transform of the frequency

response $H(\Omega)$. Using partial fractions, the frequency response becomes:

$$H(\Omega) = \frac{C_1}{(3 + j\Omega)} + \frac{C_2}{(5 + j\Omega)}$$

We calculate C_1 and C_2:

$$C_1 = \frac{15 - 2j\Omega}{(3 + j\Omega)\,(5 + j\Omega)}(3 + j\Omega)\Big|_{j\Omega=-3} = \cdots = \frac{21}{2}$$

$$C_2 = \frac{15 - 2j\Omega}{(3 + j\Omega)\,(5 + j\Omega)}(5 + j\Omega)\Big|_{j\Omega=-5} = \cdots = -\frac{25}{2}$$

Thus, the frequency response in partial-fraction form is:

$$H(\Omega) = \frac{21}{2}\frac{1}{(3 + j\Omega)} - \frac{25}{2}\frac{1}{(5 + j\Omega)}$$

Therefore, the impulse response obtained by the inverse Fourier transform is:

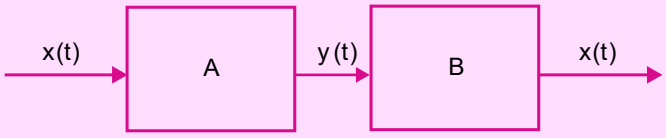
$$h(t) = \frac{1}{2}(21e^{-3t} - 25e^{-5t})u(t)$$

We obtain the same result as by solving the differential equation, using a considerably simpler procedure.

4. Inverse Systems

📖 Example 7 (*)

The continuous-time systems A and B are connected in cascade as shown below. The signal $x(t)$ enters system A, and the same signal $x(t)$ leaves system B.



If system A is LTI, determine whether system B is also LTI. What other requirements must B satisfy?

Answer: For the same signal $x(t)$ to appear at the overall output, the equivalent impulse response of the cascade must equal the unit impulse:

$$h_{eq}(t) = \delta(t)$$

Since the systems are connected in cascade, their equivalent impulse response is the convolution of their individual impulse responses. Therefore, if $h_A(t)$ and $h_B(t)$ are the impulse responses of systems A and B, respectively, we require:

$$h_A(t) * h_B(t) = \delta(t) \qquad (1)$$

Equation (1) means that system B is the inverse of system A. If $x(t)$ is applied to system A, the overall

output is:

$$x(t) = y(t) * h_B(t) = \big(x(t) * h_A(t)\big) * h_B(t)$$
$$= x(t) * \big(h_A(t) * h_B(t)\big) = x(t) * \delta(t) = x(t)$$

Thus, system B undoes the effect of system A. Equation (1) shows that B is described by an impulse response $h_B(t)$ and is therefore also LTI. In the frequency domain, equation (1) becomes:

$$H_A(\Omega)\, H_B(\Omega) = 1$$

so

$$H_B(\Omega) = \frac{1}{H_A(\Omega)}$$

Therefore, the frequency response of system B is the reciprocal of that of system A.

In the Laplace domain, we similarly have:

$$H_B(s) = \frac{1}{H_A(s)}$$

Thus, the poles of system A become zeros of system B, while the zeros of A become poles of B.

For practical implementation, system B must be causal and stable in addition to being the mathematical inverse of A. From:

$$H_B(s) = \frac{1}{H_A(s)}$$

the inverse system is stable only if the zeros of A lie in the left half of the complex plane. A system satisfying this requirement is called minimum-phase and can be inverted. Its inverse is also minimum-phase. The causal realizability of $1/H_A(s)$ must also be checked.

Comments:

- Frequency-domain inversion follows simply and directly. From $H_A(\Omega)H_B(\Omega) = 1$, we immediately obtain:

  $$H_B(\Omega) = \frac{1}{H_A(\Omega)}$$

  This result shows that frequency-domain analysis provides a clear description of system inversion.
- Mathematical invertibility does not necessarily imply practical realizability. Even if the inverse is theoretically defined, its causality and stability must also be checked. The zeros of the original system are therefore crucial, since they become poles of the inverse.

📖 Example 8

Determine whether the stable, causal system with transfer function $H(s) = (s-1)/(s+2)$ has a stable, causal inverse. Find $H_{inv}(s)$ and the impulse response of the inverse.

Answer: If the inverse exists, its transfer function is:

$$H_{inv}(s) = \frac{1}{H(s)} = \frac{s+2}{s-1}$$

The original system is causal and BIBO stable because its pole is at $s = -2$. However, it has a zero at $s = +1$, in the right half-plane. This zero becomes a pole of the inverse, making the inverse BIBO unstable. Let us examine this further. Its transfer function is:

$$H_{inv}(s) = \frac{s+2}{s-1} = 1 + \frac{3}{s-1}$$

- For $Re(s) > 1$, the impulse response is:

  $$h_{inv}(t) = \delta(t) + 3e^{t}u(t)$$

  The exponential term $e^{t}u(t)$ grows without bound, so the inverse is indeed BIBO unstable. However, it is causal because its impulse response is right-sided.

- For $Re(s) < 1$, the impulse response is:

  $$h_{inv}(t) = \delta(t) - 3e^{-t}u(t)$$

  In this case, the inverse is BIBO stable, but is noncausal because its impulse response is left-sided.

Thus, the original system is theoretically invertible as an algebraic function, but its inverse cannot be implemented as a stable, causal system. Therefore, $H(s)$ is not minimum-phase.

Comments:

- The mathematical existence of an inverse transfer function is insufficient for practical implementation. The stability and causality of the inverse must also be checked.
- A right-half-plane zero becomes a pole of the inverse, making the inverse unstable.
- This example establishes the concept of minimum phase: only when all the zeros of the original system lie in the left half-plane can we expect an inverse that is both stable and causal.

Note: Examples marked with an asterisk (*) also appear in the theory lecture slides.