UNIVERSITY OF THE PELOPONNESE – SCHOOL OF ENGINEERING
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

SIGNALS AND SYSTEMS
Solved Examples – Tutorial Sheet 8

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FOURIER SERIES

- Exponential series
- Trigonometric series
- Parseval's theorem

1. Exponential Series

📖 Example 1

Find the Type I trigonometric Fourier series of the periodic triangular signal $x(t)$, defined over one period as:

$$x(t) = A - \left(\frac{4A}{T_0}\right)|t|, \qquad |t| \le \frac{T_0}{2}$$

and extended periodically with period T_0, that is: $x(t + T_0) = x(t)$

Answer: The signal is given by:

$$x(t) = A - \left(\frac{4A}{T^0}\right)|t|, \qquad |t| \le \frac{T_0}{2}$$

The signal is even because it is defined in terms of the absolute value of time $|t|$, which is even since $|-t| = |t|$. We calculate the mean value of the signal:

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)dt = \frac{2}{T_0}\int_0^{T_0/2}\left(A - \frac{4A}{T_0}t\right)dt = \frac{2}{T_0}\left[A - \frac{4A}{T_0}t^2\right]_0^{T_0/2} = \frac{2}{T_0}\left[\frac{AT_0}{2} - \frac{AT_0}{2}\right] = 0$$

Because the signal is even, its Fourier series contains only cosine terms:

$$x(t) = \sum_{n=1}^{\infty} a_n cos(n\Omega_0 t), \quad \Omega_0 = \frac{2\pi}{T_0}$$

For the coefficients a_n, we have:

$$a_n = \frac{4}{T_0}\int_0^{T_0/2}\left[A - \left(\frac{4A}{T_0}\right)t\right]cos(n\Omega_0 t)\,dt$$

We calculate the two terms separately.

$$\frac{4A}{T_0}\int_0^{T_0/2} cos(n\Omega_0 t)\, dt = \frac{4A}{T_0}\left[\frac{sin(n\Omega_0 t)}{n\Omega_0}\right]_0^{T_0/2} = 0$$

because $sin(n\Omega_0 T_0/2) = sin(n\pi) = 0$.

For the second term, we use integration by parts:

$$\int_0^{T_0/2} t\, cos(n\Omega_0 t)\, dt = \frac{(-1)^n - 1}{n^2\Omega_0^2}$$

so:

$$a_n = \frac{16A}{T_0^2}\cdot\frac{1-(-1)^n}{n^2\Omega_0^2}$$

and since $\Omega_0 = 2\pi/T_0$, we obtain:

$$a_n = \frac{4A}{\pi^2}\cdot\frac{1-(-1)^n}{n^2}$$

Thus, for even n, we have $a_n = 0$, while for odd $n = 2m+1$, we have:

$$a_{2m+1} = \frac{8A}{\pi^2(2m+1)^2}$$

Therefore, the final expansion is:

$$x(t) = \frac{8A}{\pi^2}\cdot\sum_{m=0}^{\infty}\frac{cos\big((2m+1)\Omega_0 t\big)}{(2m+1)^2}$$

The amplitude spectrum is one-sided, contains only odd harmonics, and decays at a rate of $1/n^2$. This rapid decay reflects the greater smoothness of the triangular signal compared with a square pulse.

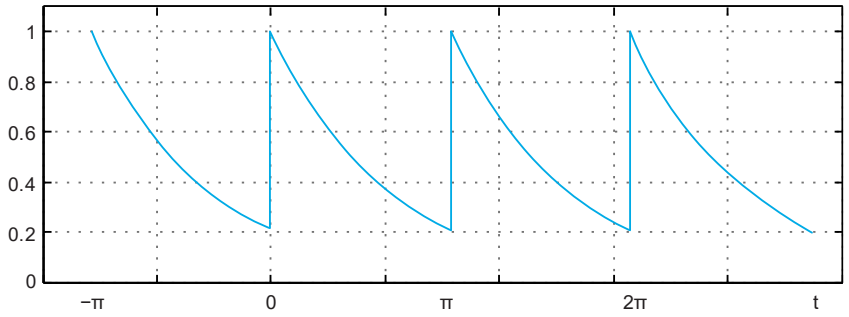
Comments

- The even symmetry of the signal makes all sine coefficients b_n zero and reduces the series to a cosine-only expansion.
- The absence of even harmonics is due to the symmetry of the triangular waveform. Only the odd harmonics $n = 1,3,5,\ldots$ appear in the expansion.
- The coefficients decay at a rate of $1/n^2$ because the triangular signal is continuous, unlike the square wave, which has discontinuities and whose coefficients decay as $1/n$.

📖 **Example 2**

Find the trigonometric and exponential Fourier series expansions of the signal $x(t)$, one period of which is given by $x_T(t) = e^{-t/2},\ 0 < t < \pi$.

Answer: The periodic exponential signal is shown in the figure. Its fundamental period is $T_0 = \pi$, so $f_0 = 1/\pi\ (Hz)$ and $\Omega_0 = 2\ (rad/sec)$.



Periodic exponential signal

We calculate the corresponding coefficients for the Fourier series forms:

- **Type I trigonometric form:**

$$\alpha_0 = \frac{1}{T_0}\int_0^{T_0} x(t)dt = \frac{1}{\pi}\int_0^{\pi} e^{-t/2}\,dt = \frac{1}{\pi}\left[-2e^{-t/2}\right]_0^{\pi} = \frac{2}{\pi}\left(1 - e^{-\pi/2}\right) \approx 0.504$$

$$\alpha_k = \frac{2}{T_0}\int_0^{T_0} x(t)\,cos(k\Omega_0 t)\,dt = \frac{2}{\pi}\int_0^{\pi} e^{-t/2}\,cos(2kt)\,dt = 1 = 0.504\frac{2}{1+16k^2}$$

$$= a_0\frac{2}{1+16k^2}$$

$$b_k = \frac{2}{T_0}\int_0^{T_0} x(t)\,sin(k\Omega_0 t)\,dt = \frac{2}{\pi}\int_0^{\pi} e^{-t/2}\,sin(2kt)\,dt = 2 = 0.504\frac{8k}{1+16k^2}$$

$$= a_0\frac{8k}{1+16k^2}$$

- **Type II trigonometric form:**

$$c_0 = a_0 = 0.504$$

$$c_k = \sqrt{a_k^2 + b_k^2} = 0.504\sqrt{\frac{4}{(1+16k^2)^2} + \frac{64k^2}{(1+16k^2)^2}} = a_0\frac{2}{\sqrt{1+16k^2}}$$

$$\theta_k = -\tan^{-1}\left(\frac{b_k}{a_k}\right) = -\tan^{-1}(4k)$$

---

1 We used the integration formula for an exponential multiplied by a cosine:

$$\int e^{ax}\cos bx\,dx = e^{ax}(a\,cos\,bx + b\,sin\,bx)\,/\,(a^2 + b^2)$$

2 We used the integration formula for an exponential multiplied by a sine:

$$\int e^{ax}\sin bx\,dx = e^{ax}(a\,sin\,bx - b\,cos\,bx)/(a^2 + b^2)$$

- **Exponential form:**

$$X_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jk\Omega_0 t}\, dt = \frac{1}{\pi}\int_0^{\pi} e^{-t/2}\, e^{-j2kt}\, dt = \frac{1}{\pi}\int_0^{\pi} e^{-(1/2+j2k)t} dt =$$

$$= -\frac{1}{\pi(1/2+j2k)}\, e^{-(1/2+j2k)t}\Big|_0^{\pi}$$

$$= \frac{1}{\pi}\frac{1-e^{-(1/2+j2k)\pi}}{1/2+j2k} = \frac{1}{\pi}\frac{1-e^{-\pi/2}}{1/2+j2k}$$

$$= \frac{2}{\pi}\frac{1-e^{-\pi/2}}{1+j4k} = \frac{0.504}{1+4jk} = \frac{\alpha_0}{1+4jk}$$

Thus:

$$|X_k| = \frac{0.504}{\sqrt{1+16k^2}},$$

$$\varphi_k = \measuredangle X_k = \tan^{-1}\left(-\frac{b_k}{a_k}\right) = -\tan^{-1}(4k)$$

Therefore, the Fourier series expansions of the periodic exponential function are:

$$x(t) = 0.504 + 0.504\sum_{k=1}^{\infty}\frac{2}{1+16k^2}\cos(2kt) + \frac{8k}{1+16k^2}\sin(2kt)$$

$$x(t) = 0.504 + 0.504\sum_{k=1}^{\infty}\frac{2}{\sqrt{1+16k^2}}\cos[2kt - tan^{-1}(4k)]$$

$$x(t) = 0.504\sum_{k=-\infty}^{\infty}\frac{1}{1+4jk}e^{j2kt}$$

Comments:

- The signal $x_T(t) = e^{-t/2}, 0 < t < \pi$ has neither even nor odd symmetry about $t = 0$. Therefore, its Fourier series generally contains both cosine and sine terms: both the coefficients a_k and b_k are nonzero.
- The mean value of the signal over one period is:

$$a_0 = \frac{2}{\pi}(1 - e^{-\pi/2})$$

and equals the coefficient X_0 of the exponential Fourier series, as expected.
- The exponential Fourier series coefficients are given by:

$$X_k = \frac{a_0}{1+j4k}$$

so their magnitude is:

$$|X_k| = \frac{|a_0|}{\sqrt{1+16k^2}}$$

- For large values of k, the coefficients decay asymptotically as $1/k$. This relatively slow decay occurs because the periodic extension of the signal has discontinuities at the period boundaries.
- Indeed, at the end of each period, the signal has the value $e^{-\pi/2}$, while at the beginning of the next period it returns to 1. Therefore, the periodic extension has a jump, which explains why the spectrum does not decay more rapidly.
- Since the signal is real, its exponential Fourier series coefficients satisfy the conjugate-symmetry relation:

$$X_{-k} = Xk^*$$

- Consequently, the amplitude spectrum is an even function and the phase spectrum is odd:
$$|X_{-k}| = |X_k|, \qquad \angle X_{-k} = -\angle X_k$$
- The phase of the coefficients is:
$$\phi_k = -\tan^{-1}(4k)$$
and is negative for $k > 0$, reflecting the relative time asymmetry of the signal within each period.
- According to the Fourier series convergence theorem, at discontinuities of the periodic extension the series converges to the average of the one-sided limits. Therefore, at $t = n\pi, n \in Z$, the Fourier series converges to:
$$\frac{1 + e^{-\pi/2}}{2}$$

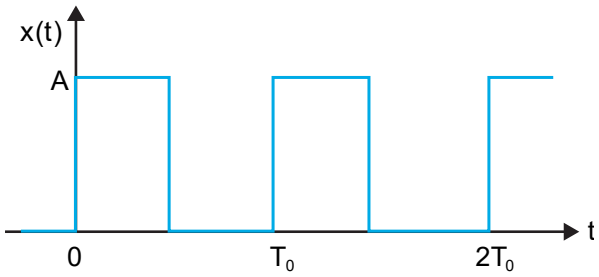
2. Trigonometric Series

Example 3

Find the Type I trigonometric Fourier series expansion of the signal $x(t)$, one period of which is:

$$x_T(t) = \begin{cases} A, & 0 < t < \dfrac{T_0}{2} \\ 0, & \dfrac{T_0}{2} < t < T_0 \end{cases}$$

Answer: From the given expression for one period $x_T(t)$ of the signal, the plot of the complete signal $x(t)$ is shown in the following figure.



Square pulse train

This is a pulse train with period T_0 and pulse duration $T_0/2$. We calculate the Type I trigonometric coefficients:

$$\alpha_0 = \frac{1}{T_0}\int_0^{T_0} x(t)dt = \frac{1}{T_0}\int_0^{T_0/2} A\,dt = \frac{A}{T_0}t\Big|_0^{T_0/2} = \frac{A}{T_0}\frac{T_0}{2} = \frac{A}{2}$$

$$\alpha_k = \frac{2}{T_0}\int_0^{T_0} x(t)\,cos(k\Omega_0 t)\,dt = \frac{2A}{T_0}\int_0^{T_0/2} cos(k\Omega_0 t)\,dt =$$

$$= \frac{2A}{T_0}\frac{1}{k\Omega_0} sin(k\Omega_0 t)\,t\Big|_0^{T_0/2} =$$

$$\frac{2A}{T_0}\frac{T_0}{2k\pi} sin\left(\frac{2k\pi}{T_0}t\right)t\Big|_0^{T_0/2} = \frac{A}{k\pi}[sin(k\pi) - sin(0)] = 0$$

The calculated result $a_k = 0$ shows that, apart from the DC component $A/2$, the signal is

expanded using only sine harmonics.

$$b_k = \frac{2}{T_0}\int_0^{T_0} x(t)\,sin(k\Omega_0 t)\,dt = \frac{2A}{T_0}\int_0^{\frac{T_0}{2}} sin(k\Omega_0 t)\,dt$$

$$= -\frac{2A}{T_0}\frac{1}{k\Omega_0}cos(k\Omega_0 t)\,t\Big|_0^{\frac{T_0}{2}} = -\frac{2A}{T_0}\frac{T_0}{2k\pi}cos\left(\frac{2k\pi}{T_0}t\right)t\Big|_0^{\frac{T_0}{2}}$$

$$= -\frac{A}{k\pi}[cos(k\pi) - cos(0)] = -\frac{A}{k\pi}[cos(k\pi) - 1]$$

$$= \frac{A}{k\pi}[1 - cos(k\pi)] = \frac{A}{k\pi}[1 - (-1)^k]$$

Thus:

$$b_k = \begin{cases} \frac{2A}{k\pi}, & k = \text{odd} \\ 0, & k = \text{even} \end{cases}$$

Therefore, the Type I trigonometric Fourier series expansion of this square pulse train is:

$$x(t) = \frac{A}{2} + \frac{2A}{\pi}\sum_k \frac{1}{k}sin(k\Omega_0 t)\,, \quad k = 1,3,5,\ldots$$

Comments:

- The signal is a rectangular pulse train with period T_0, amplitude A, and a 50% duty cycle, since the pulse duration is $\tau = T_0/2$.
- The mean value of the signal is $a_0 = A/2$, as expected, since the signal equals A for half the period and 0 for the remaining half.
- If we subtract the mean value from the signal, the resulting function $x(t) - A/2$ has odd symmetry. Therefore, apart from the DC component A/2, the Fourier expansion contains only sine terms. This explains why $a_k = 0$.
- The coefficients:

  $$b_k = \frac{A}{k\pi}[1 - (-1)^k]$$

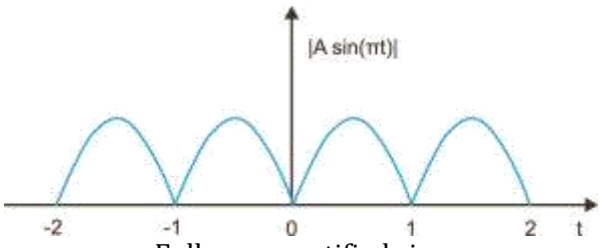
  are zero for even values of k. Therefore, only odd harmonics appear in the expansion. This is characteristic of rectangular pulses with a 50% duty cycle.
- The signal in this example can be viewed as a time-shifted version of the corresponding even pulse with amplitude A and duration T_0, shifted by $t_0 = T_0/4$. This time shift changes only the phases of the harmonics, leaving their amplitudes unchanged. This explains why the amplitude spectrum remains the same as in the previous example, while the form of the time-domain expansion changes.
- According to the Fourier series convergence theorem, at discontinuities the series converges to the average of the one-sided limits. Thus, the Fourier series converges to A/2 at:

  $$t = nT_0 \quad \text{and} \quad t = nT_0 + \frac{T_0}{2}, \qquad n \in Z$$

📖 Example 4 (*)

Find the Fourier series expansion of the full-wave rectified sinusoidal signal shown in the following figure, and plot its one-sided and two-sided amplitude spectra and its phase spectrum.

Answer: The full-wave rectified sine is written as $x(t) = |Asin(\pi t)|$. Its fundamental period is $T_0 = 1\ sec$, so the fundamental frequency is $f_0 = 1\ (Hz)$ and the fundamental angular frequency is $\Omega_0 = 2\pi\ (rad/sec)$. Full-wave rectification halves the period compared with the original sine wave.



Full-wave rectified sine

We calculate the corresponding coefficients for the Fourier series forms:

- **Type I trigonometric form:**

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt = \int_0^1 A\sin(\pi t)\,dt = \left[-\frac{A}{\pi}\cos(\pi t)\right]_0^1 = -\frac{A}{\pi}[\cos(\pi)-\cos(0)] = \frac{2A}{\pi}$$

$$a_k = \frac{2}{T_0}\int_0^{T_0} x(t)\,cos(k\Omega_0 t)\,dt = 2A\int_0^1 \sin(\pi t)\,cos(2k\pi t)\,dt = \ldots 3$$

$$= 2A\left[-\frac{\cos(\pi-2k\pi)}{2(\pi-2k\pi)}t - \frac{\cos(\pi+2k\pi)}{2(\pi+2k\pi)}t\right]_0^1$$

$$= 2A\left[\frac{\cos(2k\pi-\pi)}{2(2k\pi-\pi)} - \frac{\cos(2k\pi+\pi)}{2(2k\pi+\pi)}\right] = \cdots 4$$

$$= 2A\left[-\frac{1}{4k\pi-2\pi} + \frac{1}{4k\pi+2\pi}\right] = 2A\left[\frac{-4\pi}{(4k\pi)^2-(2\pi)^2}\right]$$

$$= -\frac{4A}{\pi\left(4k^2-1\right)} = \frac{4A}{\pi\left(1-4k^2\right)}$$

The coefficients b_k are calculated using the following expression and are zero because the function is even:

$$b_k = \frac{2}{T_0}\int_0^{T_0} x(t)sin(k\Omega_0 t)\,dt = \cdots = 0$$

- **Type II trigonometric form:**

$$c_0 = a_0 = \frac{2A}{\pi}$$

$$c_k = \sqrt{a_k^2 + b_k^2} = |\alpha_k| = \frac{4A}{\pi(4k^2-1)}$$

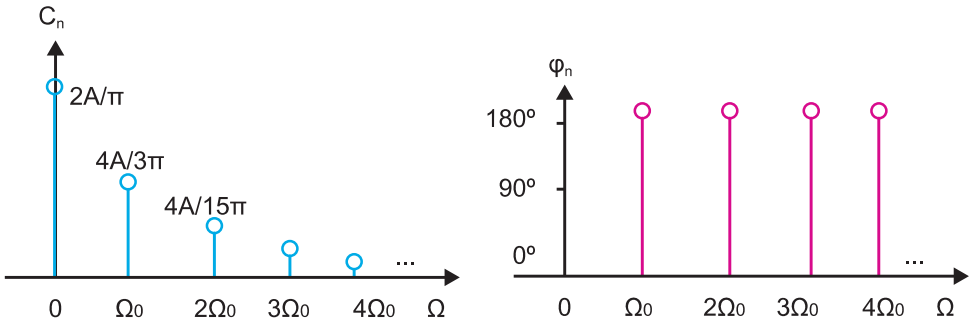
$$\theta_k = \tan^{-1}\left(\frac{b_k}{a_k}\right) = \pi \quad (180^0), \qquad k \geq 1$$

From the Type II trigonometric form, we can plot the one-sided amplitude spectrum $c_k =$

3 We used the integration formula for a sine multiplied by a cosine:
$\int sinPx\,cosQx\,dx = -\frac{\cos(P-Q)x}{2(P-Q)} - \frac{\cos(P+Q)x}{2(P+Q)}$ where $P = \pi$ and $Q = 2k\pi$

4 We used the identities: $\cos(2k\pi-\pi) = -1$ and $\cos(2k\pi+\pi) = -1,\ \forall t$

$c_k(\Omega_k)$ and the phase spectrum $\theta_k = \theta_k(\Omega_k)$:



(a) One-sided amplitude spectrum and (b) Phase spectrum

Therefore, the Type II trigonometric expansion of $x(t)$ is:

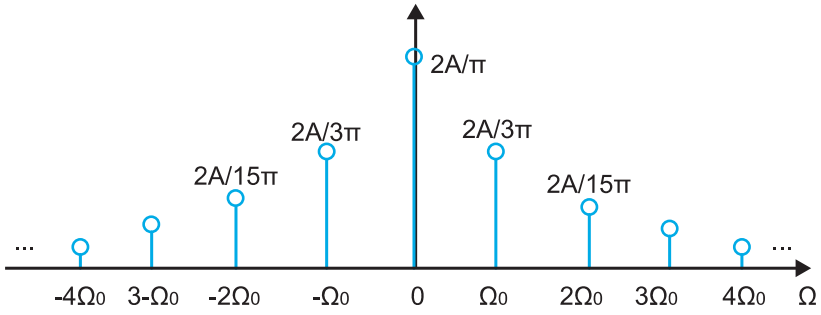
$$x(t) = \frac{2A}{\pi} + \frac{4A}{\pi}\sum_{k=1}^{\infty}\frac{\cos(k\Omega_0 t)}{(1-4k^2)}$$

- **Exponential form:**

$$X_0 = \alpha_0 = \frac{2A}{\pi} \quad \text{and} \quad X_k = \frac{1}{2}(\alpha_k - jb_k) = \frac{\alpha_k}{2} = \frac{2A}{\pi(4k^2-1)}, \qquad k \neq 0$$

$$\theta_k = \measuredangle X_k = -\tan^{-1}\left(\frac{b_k}{a_k}\right) = \pi \quad (180^0), \qquad k \neq 0$$

From the exponential form, we can construct the two-sided amplitude spectrum $|X_k| = |X_k(\Omega_k)|$ and the phase spectrum $\theta_k = \theta_k(\Omega_k)$. The phase spectrum is the same as in Figure 4.12(b). The two-sided amplitude spectrum is shown below:



(c) Two-sided amplitude spectrum

Finally, the Fourier series expansions of the periodic exponential function are:

$$x(t) = \frac{2A}{\pi} + \sum_{k=1}^{\infty}\frac{4A}{\pi(1-4k^2)}\cos(k\Omega_0 t) \xrightarrow{\Omega_0 = 2\pi} x(t) = \frac{2A}{\pi} + \frac{4A}{\pi}\sum_{k=1}^{\infty}\frac{\cos(2\pi k t)}{(1-4k^2)}$$

$$x(t) = \frac{2A}{\pi} + \sum_{k=1}^{\infty}\frac{4A}{\pi(4k^2-1)}\cos(k\Omega_0 t + \pi) \xrightarrow{\Omega_0 = 2\pi} x(t) = \frac{2A}{\pi} + \frac{4A}{\pi}\sum_{k=1}^{\infty}\frac{\cos(2\pi k t)}{(1-4k^2)}$$

$$x(t) = \frac{2A}{\pi} + \sum_{k=-\infty}^{\infty}\frac{2A}{\pi(1-4k^2)}e^{j2\pi k t}, \qquad k \neq 0$$

Comments:

- Full-wave rectification halves the period of the original sine wave, doubling the fundamental frequency of the signal. The spectrum contains only a DC component and harmonics of the frequency $f_0 = 1\ Hz$.
- The full-wave rectified sinusoidal signal $x(t) = |Asin(\pi t)|$ is even, since $x(-t) = x(t)$. Therefore, its Fourier series contains only cosine terms, and hence $b_k = 0$.
- The mean value of the signal is $\alpha_0 = 2A/\pi$, showing that the full-wave rectified sine contains a strong DC component. This component arises precisely because the signal takes only nonnegative values after rectification.
- The coefficients:
$$\alpha_k = \frac{4A}{\pi(1-4k^2)}$$
are negative for every $k \geq 1$. Therefore, in the Type II trigonometric form, the phase of the harmonics is $\theta_k = \pi$. This reflects the fact that each harmonic has a negative cosine coefficient.
- The magnitude of the spectral coefficients decreases as the index k increases. More specifically:
$$|a_k| \sim \frac{1}{k^2} \text{ for large } k$$
This behavior shows that the higher harmonics decay relatively quickly, so the signal can be approximated well using only a few terms of its Fourier series.
- Since the signal is real and even, its exponential Fourier series coefficients are real and satisfy $X_{-k} = X_k$. Thus, the two-sided amplitude spectrum is even and the phase spectrum is symmetric about the frequency axis.
- This example shows that rectification changes both the time-domain waveform and its spectral structure, introducing a DC component and new harmonic components.

📖 **Example 5**

Calculate the Fourier series of $x(t) = A\max\{sin(\Omega_0 t), 0\}$ (a half-wave rectified sinusoidal signal) and identify its main spectral components.

Answer: Letting $\theta = \Omega_0 t$, the signal is written as:

$$x(t) = \begin{cases} A\ sin\theta, & 0 < \theta < \pi \\ 0, & \pi < \theta < 2\pi \end{cases}$$

We can write the signal as the sum of an odd component and an even component:

$$x(t) = \frac{A}{2} sin(\Omega_0 t) + \frac{A}{2}|sin(\Omega_0 t)|$$

The full-wave rectified sine $|sin(\Omega_0 t)|$ has a known expansion in cosine components at even harmonics:

$$|sin(\Omega_0 t)| = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{cos(2n\Omega_0 t)}{4n^2 - 1}$$

Multiplying by $A/2$ and adding the term $A/2 \cdot sin(\Omega_0 t)$, we obtain:

$$x(t) = \frac{A}{\pi} + \frac{A}{2} sin(\Omega_0 t) - \frac{2A}{\pi} \sum_{n=1}^{\infty} \frac{cos(2n\Omega_0 t)}{4n^2 - 1}$$

Thus, the coefficients in the Type I trigonometric form are:

$$a_0 = \frac{A}{\pi}, \quad a_{2n} = -\frac{2A}{\pi(4n^2 - 1)},$$
$$a_{2n-1} = 0 \text{ for } n \geq 1, \quad b_1 = \frac{A}{2}, \quad b_n = 0 \text{ for } n \neq 1$$

The Type II trigonometric form is:

$$x(t) = \frac{A}{\pi} + \frac{A}{2} sin(\omega_0 t) + \sum_{n=1}^{\infty} \frac{2A}{\pi(4n^2 - 1)} cos(2n\Omega_0 t + \pi)$$

The one-sided spectrum contains a DC component A/π, a fundamental harmonic at frequency Ω_0 with amplitude $A/2$ and zero phase relative to the sine, and only even cosine harmonics $2\Omega_0$, $4\Omega_0$, $6\Omega_0$, ... with coefficients a_{2n} and phase π.

Comments

- The half-wave rectified sine is neither even nor odd, so both cosine and sine terms appear in its Fourier expansion.
- The decomposition:

$$x(t) = \frac{A}{2} \sin(\Omega_0 t) + \frac{A}{2} |\sin(\Omega_0 t)|$$

  simplifies the calculation by using the known expansion of the full-wave rectified sine.
- The fundamental harmonic comes exclusively from the odd term, while the even harmonics come from the rectified even component.

3. Parseval's Theorem

📖 **Example 6**
Using the two-sided amplitude spectrum of the previous example, calculate the power of each harmonic term, both as an absolute value and as a percentage of the total power.

Answer: To find the absolute power of each harmonic term, we calculate the power-spectrum

coefficients $|X_k|^2,\ k = 0, \pm1, \pm2, \ldots$. The power spectrum is obtained by squaring the amplitude spectrum:

- For $k = 0$, we have: $$P_0 = |X_0|^2 = \left(\frac{2A}{\pi}\right)^2 = \frac{4A^2}{\pi^2}$$
- For $k = 1$, we have: $$P_1 = |X_1|^2 + |X_{-1}|^2 = 2\left(\frac{2A}{3\pi}\right)^2 = \frac{8A^2}{9\pi^2}$$
- For $k = 2$, we have: $$P_2 = |X_2|^2 + |X_{-2}|^2 = 2\left(\frac{2A}{15\pi}\right)^2 = \frac{8A^2}{225\pi^2}$$

We calculate the remaining harmonic terms similarly. To find the percentage of power carried by each harmonic term, we need the total power. Parseval's theorem gives:

$$P_{total} = \frac{1}{T}\int_0^T x^2(t)\,dt = \frac{1}{1}\int_0^1 A^2 \sin^2(2\pi t)\,dt = \frac{A^2}{2}$$

Therefore, the required percentage of power carried by the DC term is:

$$P'_0 = \frac{P_0}{P_{total}}(\%) = \frac{\frac{4A^2}{\pi^2}}{\frac{A^2}{2}}(\%) = \frac{8}{\pi^2} \cong 81.06\%$$

Similarly, we calculate the percentage of power carried by the remaining terms:

$$P'_k = \frac{P_k}{P_{total}} = \frac{2|X_k|^2}{P_{total}},\quad n = 1,2\ldots$$

The first harmonic carries approximately:

$$P'_1 = \frac{16}{9\pi^2} \approx 18.01\%$$

The second harmonic:

$$P'_2 = \frac{16}{225\,\pi^2} \approx 0.72\%$$

The third harmonic:

$$P'_3 = \frac{16}{1225\,\pi^2} \approx 0.13\%$$

Comments:

- The power of the full-wave rectified sinusoidal signal is not distributed uniformly among all harmonics. Instead, 99.07% is concentrated in the DC component (81.06% of the total power) and the first harmonic (18.01% of the total power). The higher harmonics contribute progressively less.
- This result shows that the signal's power spectrum is strongly concentrated at low frequencies. Therefore, in terms of power, the signal can be described adequately using even a small number of Fourier series terms.
- Since the signal is real and even, its exponential Fourier series coefficients are symmetric about the index $k = 0$, so the components at frequencies $+k\Omega_0$ and $-k\Omega_0$ carry equal power. Therefore, the total power of the kth harmonic for $k \geq 1$ is calculated by adding the terms $|X_k|^2$ and $|X_{-k}|^2$.

- The total power calculated in the time domain agrees with the sum of the individual spectral components, according to Parseval's theorem. Thus, this example illustrates the equivalence between the time-domain and frequency-domain descriptions of power.
- Finally, the rapid decrease in the power of the higher harmonics shows that high frequencies contribute little to the overall waveform. This explains why the first few harmonics usually provide a good approximation of the rectified sine.

Note: Examples marked with an asterisk (*) also appear in the theory lecture slides.