UNIVERSITY OF THE PELOPONNESE – SCHOOL OF ENGINEERING
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

SIGNALS AND SYSTEMS
Solved Examples – Tutorial Handout 1

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INTRODUCTION TO CONTINUOUS-TIME SIGNALS

- Characteristic Signal Parameters
- Transformations of the Independent Variable
- Types of Continuous-Time Signals

1. Characteristic Signal Parameters

📖 Example 1

Determine which of the following are energy signals, power signals, or neither:

(a) $x(t) = e^{at},\ a > 0,\ t \geq 0$ (b) $x(t) = e^{at},\ a < 0,\ t \geq 0$

Answer: (a) Since the signal has infinite duration and unbounded amplitude, both its energy and its power tend to infinity. Therefore, it is neither an energy signal nor a power signal.

(b) The signal has infinite duration and bounded amplitude. Its energy is:

$$E_x = \int_{-\infty}^{+\infty} x^2(t)dt = \int_{0}^{+\infty} e^{2\alpha x}\, dt = \frac{1}{2a}\left[\lim_{t\to\infty} e^{2at} - 1\right] = -\frac{1}{2a} < \infty$$

The signal's power is zero. Therefore, it is an energy signal and a power signal.

📖 Example 2

Determine the energy of the signal:

$$x(t) = \begin{cases} e^{-2t}\sin t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Answer: Using the trigonometric identity:

$$sin^2 t = \frac{1}{2}[1 - cos(2t)]$$

we obtain:

$$x^2(t) = (e^{-2t}\sin t)^2 = e^{-4t}\sin^2 t = \frac{1}{2}\, e^{-4t}[1 - \cos(2t)]$$

Substituting the expression above into the defining equation for energy:

$$E_x = \int_{-\infty}^{+\infty} x^2(t)\, dt$$

and since the given signal is defined only for $t \geq 0$, we obtain:

$$E_x = \int_{-\infty}^{+\infty} x^2(t)\,dt = \frac{1}{2}\int_0^{\infty} e^{-4t}[1-\cos(2t)]dt = \frac{1}{2}\int_0^{\infty} e^{-4t}dt - \frac{1}{2}\int_0^{\infty} e^{-4t}\cos(2t)dt$$

The first integral is evaluated as follows:

$$\int_0^{\infty} e^{-4t}dt = \left[-\frac{1}{4}e^{-4t}\right]_0^{\infty} = \frac{1}{4}$$

The second integral is evaluated by repeated integration by parts, giving:[1]

$$\int_0^{\infty} e^{-4t}\,cos(2t)dt = \frac{1}{5}$$

Substituting these integral values into the defining equation for energy, we obtain:

$$E_x = \frac{1}{40}$$

Therefore, the given signal $x(t)$ is an energy signal.

📖 Example 3
Calculate the energy and power of the signal $x(t) = \cos(\pi t/2 + \pi/4)$.

Answer: The energy is calculated from:

$$E_x = \int_{-\infty}^{+\infty} x^2(t)dt = \int_{-\infty}^{+\infty} \cos^2\left(\frac{\pi t}{2}+\frac{\pi}{4}\right)dt \to \infty$$

Therefore, the given signal has infinite energy. Its power is calculated using the signal's symmetry and setting $T = NT_0$:

$$P_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x^2(t)dt = \lim_{T\to\infty}\frac{2}{2T}\int_0^{T}\cos^2\left(\frac{\pi t}{2}+\frac{\pi}{4}\right)dt$$
$$= \lim_{N\to\infty}\frac{1}{NT_0}\int_0^{NT_0}\cos^2\left(\frac{\pi t}{2}+\frac{\pi}{4}\right)dt = \lim_{N\to\infty}\frac{1}{NT_0}N\int_0^{T_0}\cos^2\left(\frac{\pi t}{2}+\frac{\pi}{4}\right)dt$$
$$= \frac{1}{T_0}\int_0^{T_0}\cos^2\left(\frac{\pi t}{2}+\frac{\pi}{4}\right)dt = \frac{1}{2T_0}\int_0^{T_0}\left[\cos\left(\pi t+\frac{\pi}{2}\right)+1\right]dt$$
$$= \frac{1}{8}\int_0^{4}\cos\left(\pi t+\frac{\pi}{2}\right)dt + \frac{1}{8}\int_0^{4}dt = 0+\frac{1}{2} = \frac{1}{2}$$

In the solution, we used the trigonometric identity:

$$\cos^2\left(\frac{\pi t}{2}+\frac{\pi}{4}\right) = \frac{1}{2}\left[\cos\left(\pi t+\frac{\pi}{2}\right)+1\right]$$

[1] Integration by parts: $\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$

📖 Example 4
Determine whether the signal $x(t) = t\,u(t)$ is an energy signal, a power signal, or neither.

Answer: The signal can be written as:

$$x(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}$$

This is a causal signal with infinite duration and unbounded amplitude. Its energy is:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_0^{\infty} t^2\, dt$$

but

$$\int_0^{\infty} t^2\, dt = \infty$$

so the signal is not an energy signal.

The average power is:

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt$$

Since $x(t) = 0$ for $t < 0$, we obtain:

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_0^{T} t^2\, dt$$

We calculate:

$$\int_0^{T} t^2\, dt = \frac{T^3}{3}$$

hence:

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \cdot \frac{T^3}{3} = \lim_{T\to\infty} \frac{T^2}{6} = \infty$$

Thus, the average power is not finite, so the signal is not a power signal. Therefore, the signal $x(t) = t\,u(t)$ is neither an energy signal nor a power signal.

Comments:

- Infinite duration alone is not sufficient to classify a signal as a power signal. If its amplitude grows without bound, the criterion of finite average power may also fail.
- Signals that are neither energy signals nor power signals are not uncommon. They often occur in ideal mathematical models when a signal grows very rapidly.
- Despite the factor $1/2T$ in the definition of average power, the cubic growth of the integral dominates, and the power becomes infinite.
- The form $t\,u(t)$ is particularly useful because it also provides a simple counterexample to the incorrect intuition that every causal signal of infinite duration is a power signal.

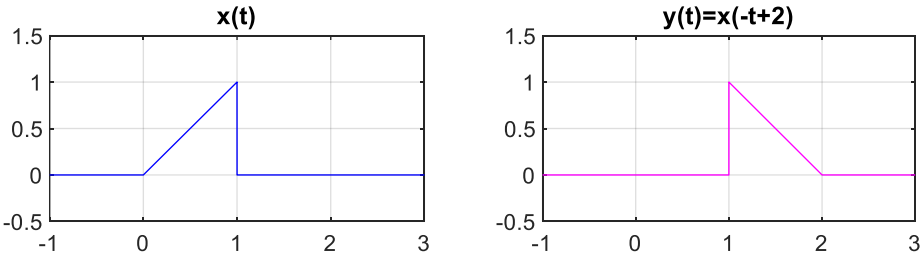
2. Transformations of the Independent Variable

📖 Example 5
Calculate and sketch the signal $y(t) = x(-t + 2)$, given that the signal $x(t)$ is defined by:

$$x(t) = \begin{cases} t, & 0 \leq t < 1 \\ 0, & \text{otherwise} \end{cases}$$

Answer: The signal $y(t)$ clearly results from time reversal and a time shift, but it is not immediately clear which transformation was applied first. It is also unclear whether the time shift represents a delay or an advance.

Since the signal can be written as $x(-t+2) = x\big(-(t-2)\big)$, it follows that the signal $x(t)$ was first delayed by two time units and then reversed in time.



(a) Signal $x(t)$, (b) Signal $y(t) = x(-t+2)$

If we are unsure about the order of the transformations, the safest approach is to calculate the values of the signal $y(t)$ and then sketch it. We obtain:

$\boldsymbol{t}$	$\boldsymbol{y(t) = x(-t+2)}$
-1	$x(3) = 0$
-0.5	$x(2.5) = 0$
0	$x(2) = 0$
1	$x(1) = 1$
1.5	$x(0.5) = 0.5$
2	$x(0) = 0$

Comment: The expression $-t+\tau$ corresponds to time reversal and delay, whereas the expression $-t-\tau$ corresponds to time reversal and advance.

📖 Example 6

Given the signal:

$$x(t) = \begin{cases} t+1, & -1 \le t < 0 \\ 1, & 0 \le t < 1 \\ 2-t, & 1 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Calculate the signal $y(t) = x(2-3t)$ and determine the sequence of transformations required to obtain it from $x(t)$.

Answer: To determine the order of the transformations, we rewrite the expression $y(t) = x(2-3t)$ as:

$$y(t) = x\left[-3\left(t - \frac{2}{3}\right)\right]$$

The correct interpretation is therefore: (a) reverse $x(t)$ in time about $t = 0$, (b) compress time by a factor of 3, and (c) apply a time delay of $2/3$. To derive $y(t)$ analytically, set $\tau = 2-3t$ and determine the values of t for which τ lies in the intervals defining $x(\tau)$.

- For $-1 \le \tau < 0$, we obtain:

$$-1 \le 2-3t < 0$$

The left inequality gives $t \le 1$, while the right inequality gives $t > 2/3$. Thus:

$$\frac{2}{3} < t \le 1$$

Therefore:

$$y(t) = \tau + 1 = (2-3t) + 1 = 3-3t$$

- For $0 \le \tau < 1$, we obtain:

$$0 \le 2 - 3t < 1$$

hence

$$\frac{1}{3} < t \le \frac{2}{3}$$

Therefore:

$$y(t) = 1$$

- For $1 \le \tau \le 2$, we obtain:

$$1 \le 2 - 3t \le 2$$

hence

$$0 \le t \le \frac{1}{3}$$

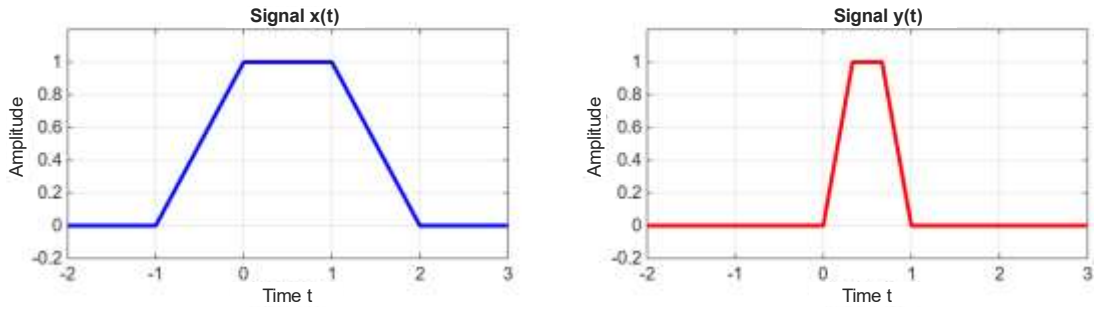
Therefore:

$$y(t) = 2 - \tau = 2 - (2 - 3t) = 3t$$

Finally:

$$y(t) = \begin{cases} 3t, & 0 \le t \le 1/3 \\ 1, & 1/3 < t \le 2/3 \\ 3 - 3t, & 2/3 < t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

The signal is a copy of the original, compressed in time by a factor of 3, reversed in time, and shifted to occupy the interval [0,1].



(a) Signal $x(t)$, (b) Signal $y(t)$

Comments:

- This example shows that the expression $x(at + b)$ should not be split into scaling and shifting operations before it has been rearranged algebraically. The form $x[-3(t - 2/3)]$ reveals the correct sequence of operations.
- A negative coefficient of t indicates time reversal, while the value $|a| = 3$ requires time compression by a factor of 3.
- Using inequalities to determine the defining intervals is a reliable general method, especially for piecewise-defined signals.

3. Types of Continuous-Time Signals

Example 7
For the signal $x(t) = cos(2\pi t + \theta), -\infty < t < +\infty$, determine the values of θ for which the signal $x(t)$ has even and odd symmetry, respectively.

Answer: An even signal satisfies $x(-t) = x(t)$. Therefore, the given signal must satisfy:

$$cos(2\pi t + \theta) = cos(-2\pi t + \theta) = cos(2\pi t - \theta)$$

Thus, we require $\theta = -\theta$, namely $\theta = 0$ or $\theta = \pi$, giving:

$$x_1(t) = cos(2\pi t) \text{ or } cos(2\pi t + \pi) = -\cos(2\pi).$$

Indeed, the cosine is an even signal.

For the signal to be odd, we require $x(-t) = -x(t)$. Therefore:

$$cos(2\pi t + \theta) = -cos(-2\pi t + \theta) = cos(-2\pi t + \theta \pm \pi) = cos(2\pi t - \theta \mp \pi)$$

Thus, we require $\theta = -\theta \mp \pi$, namely $\theta = \mp\frac{\pi}{2}$, giving:

$$x_2(t) = cos\left(2\pi t - \frac{\pi}{2}\right) = sin(2\pi t) \text{ and } cos\left(2\pi t + \frac{\pi}{2}\right) = -sin(2\pi t).$$

Indeed, the sine is an odd signal.

When $\theta = \pi/4$, the signal $cos(2\pi t + \pi/4)$ is neither even nor odd.

Example 8
Determine whether the following signals have even symmetry, odd symmetry, or neither:

(a) $x(t) = 4t$ (b) $x(t) = e^{-|t|}$
(c) $x(t) = \sin(2t)$ (d) $x(t) = sin(2t - \pi/2)$

Answer: (a) We have $x(-t) = 4(t) = -4t = -x(t)$, so it is an odd function.
(b) We have $x(-t) = e^{-|-t|} = e^{-|t|} = x(t)$, so it is an even function.
(c) We have $x(-t) = sin\big(2(-t)\big) = -sin(2t) = -x(t)$, so it is an odd function.
(d) We have $x(t) = sin\left(2t - \frac{\pi}{2}\right) = -\cos(2t)$ and also $x(-t) = sin\left(-2t - \frac{\pi}{2}\right) = -sin\left(2t + \frac{\pi}{2}\right) = -\cos(2t) = x(t)$, so it is an even function.

Example 9
Examine the following signals for periodicity and calculate the fundamental period T_0 of each:

(a) $x(t) = 3\cos(5t + \pi/4)$ (b) $x(t) = 2e^{j(\pi t - 1)}$

Answer: By definition, a signal is periodic if $x(t + T) = x(t)$ holds for every time t, where T is a positive number.

We therefore need to determine whether there is a positive number T for which this relation holds. For (a), we obtain:

$$x(t + T) = x(t) \Rightarrow 3\cos\left(5t + 5T + \frac{\pi}{4}\right) = 3\cos\left(5t + \frac{\pi}{4}\right)$$

We know that if $\cos\varphi = \cos\theta$, then $\varphi \pm \theta = 2k\pi$. Therefore:

$$\left(5t+5T+\frac{\pi}{4}\right)+\left(5t+\frac{\pi}{4}\right)=2k\pi \Rightarrow T=\frac{2k\pi}{5}-2t-\frac{\pi}{10}$$

or

$$\left(5t+5T+\frac{\pi}{4}\right)-\left(5t+\frac{\pi}{4}\right)=2k\pi \Rightarrow T=\frac{2k\pi}{5}$$

The signal is periodic, with fundamental period (for $k=1$) $T_0=2\pi/5$ and fundamental frequency $f_0=5/2\pi$.

(b) Proceeding similarly, we obtain:

$$x(t+T)=x(t) \Longrightarrow 2e^{j(\pi t+\pi T-1)}=2e^{j(\pi t-1)}$$
$$\Longrightarrow 2e^{j\pi T}e^{j(\pi t-1)}=2e^{j(\pi t-1)} \Longrightarrow e^{j\pi T}=1$$
$$\Longrightarrow \cos(\pi T)+jsin(\pi T)=1 \Longrightarrow \pi T=2k\pi \Longrightarrow T=2k$$

The signal is periodic, with fundamental period $T_0=2$ and fundamental frequency $f_0=1/2$.

📖 Example 10

(a) Determine whether the signal $y(t)=x(2t)$ is periodic, where $x(t)$ is given by:

$$x(t)=\cos(2t)+\sin(5t)$$

(b) If it is periodic, determine its fundamental period.

Answer: (a) Substituting $t=2t$, we obtain:

$$y(t)=\cos(4t)+\sin(10t)$$

The new signal is therefore the sum of two sinusoids, with angular frequencies $\Omega_1=4$ and $\Omega_2=10$. Their respective periods are:

$$T_1=\frac{2\pi}{4}=\frac{\pi}{2}, \qquad T_2=\frac{2\pi}{10}=\frac{\pi}{5}$$

The ratio of the periods is rational:

$$\frac{T_1}{T_2}=\frac{\pi/2}{\pi/5}=\frac{5}{2}$$

Therefore, $y(t)$ is periodic.

(b) To determine the fundamental period, set:

$$T_0=k_1\frac{\pi}{2}=k_2\frac{\pi}{5}$$

that is:

$$5k_1=2k_2$$

The smallest solution is $k_1=2$ and $k_2=5$, giving:

$$T_0=\pi$$

Comments:

- The original signal $x(t)=cos(2t)+sin(5t)$ is also periodic, with fundamental period 2π. As expected, time compression by a factor of 2 has halved the period.
- Time scaling directly affects the angular frequencies and, therefore, the periods of a signal's components. It changes both the signal's form and the fundamental properties of its periodicity.
- The substitution $t=2t$ doubles the frequencies and halves the periods. This inverse relationship between time and frequency is fundamental and will reappear later in Fourier theory and sampling.

Note: Examples marked with an asterisk (*) also appear in the lecture slides.