

UNIVERSITY OF THE PELOPONNESE – SCHOOL OF ENGINEERING
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

Signals and Systems

Lecture 1: Introduction to Continuous-Time Signals

Michalis Paraskevas
Professor

Lecture Contents

1. Objectives of Signal Processing
2. Signal Classification by Time and Amplitude
3. Characteristic Signal Parameters
4. Signal Classification by Energy and Power
5. Transformations of the Independent Variable
6. Classes of Continuous-Time Signals

1. Objectives of Signal Processing

Objectives of Signal Processing

The objectives of signal processing are:

- To develop mathematical models that adequately describe the various classes of signals carrying useful information.
- To analyze the systems that process and transmit signals.
- To design optimal systems that minimize the effect of noise on signals.

2. Signal Classification by Time and Amplitude

Signal Classes

In terms of the **time** variable:

- **Continuous-time** signal: a signal $x(t)$ defined for **every value of t** in the time interval (a, b).
- **Discrete-time** signal: a signal defined only at specific instants in time.
  Discrete-time signals are represented by **sequences** $\{x(n)\}$. The value of the sequence $\{x(n)\}$ at time n_0 is the scalar quantity $x(n_0)$.

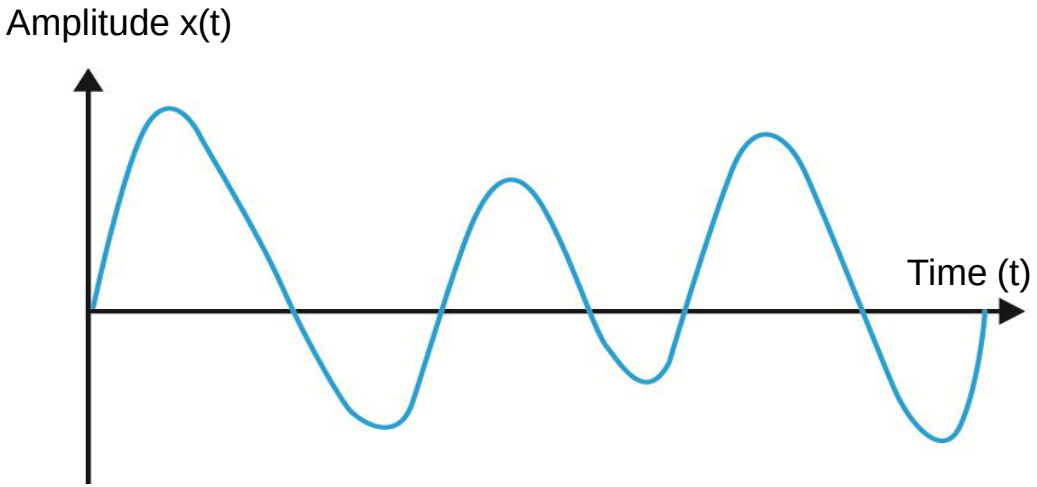
In terms of the **amplitude** variable:

- **Continuous-valued:** a signal that can take any value within a range.
- **Discrete-valued:** a signal that takes values from a finite set.

Continuous-Time Signals

Continuous-time signals are divided into two classes:

- **Analog signals**, or signals with continuous time and amplitude, in which both the independent variable (t) and the dependent variable (signal amplitude) take continuous (real) values, e.g. $x(t) = 6t$.



Continuous-Time Signals

- **Discrete-valued continuous-time signals**, in which the dependent variable takes discrete values, e.g.

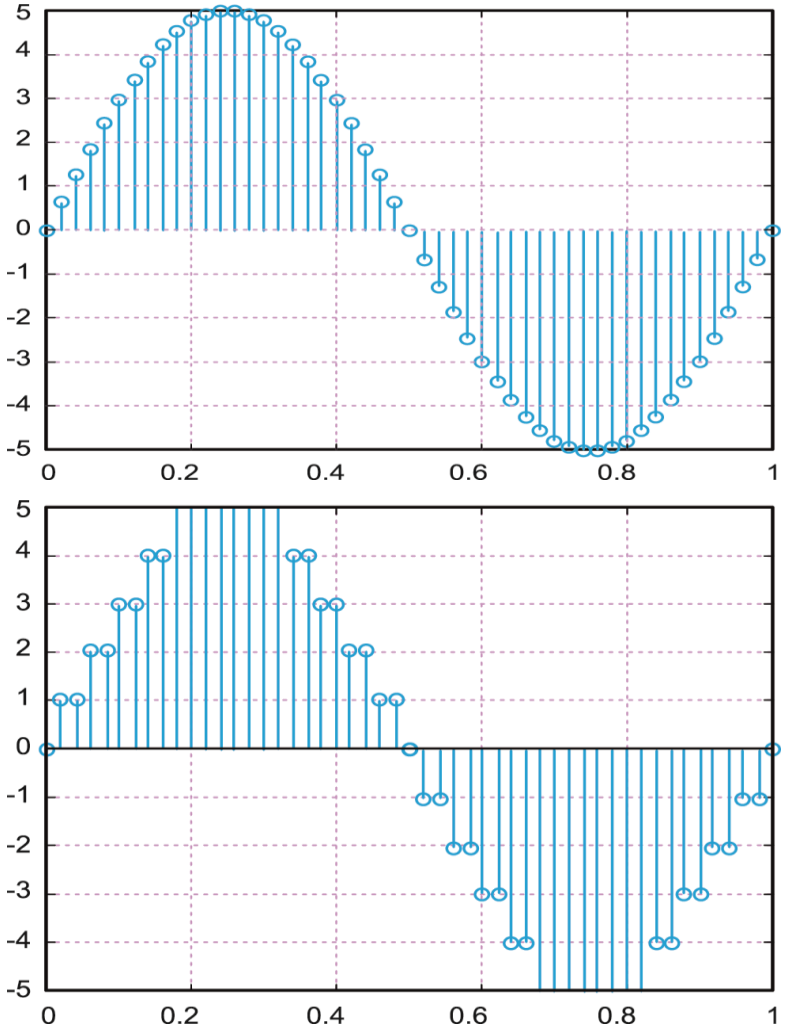
$$x(t) = \begin{cases} 0, & 0 \leq t \leq 1 \\ 1, & 1 < t \leq 2 \\ 2, & \text{otherwise} \end{cases}$$

Discrete-Time Signals

Discrete-time signals are divided into two classes:

- **Continuous-valued discrete-time signals**, in which the dependent variable (amplitude) takes continuous values, e.g. $x(t) = \cos(n\,t)$, where n is a natural number.
- **Discrete-valued discrete-time signals**, or **digital** signals, in which both the independent and the dependent variables take discrete values.

  The discrete-time signal $\{x(n)\}$ takes values from the set {-3, -2, -1, 0, 1, 2, 3}.



Common Signal Classes

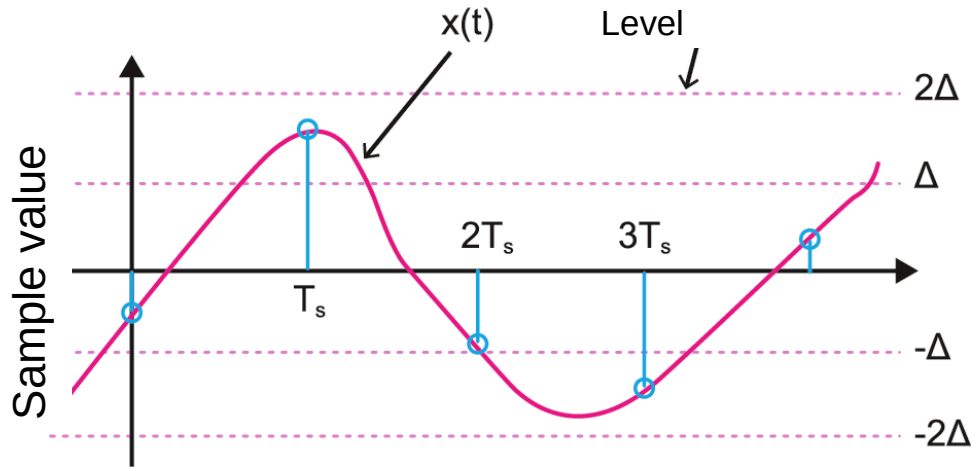
- **Analog signal**: a signal with continuous time and amplitude, in which both the independent variable (t) and the dependent variable (signal amplitude) take continuous (real) values, e.g. $x(t) = 6t$
- **Digital signal:** a discrete-time, discrete-valued signal.
- Signals usually occur in nature in analog form, e.g. speech and music. Advances in **digital technology**, however, allow digital signals to be processed, transmitted and stored more efficiently.
- An analog signal is converted into a digital signal through **sampling**, **quantization** and **encoding.**

Analog-to-Digital Signal Conversion

Most natural signals are analog (speech, music, biomedical, seismic, meteorological, etc.). Digital form makes them easier to process, transmit and store.

Analog signal > Sampling > Quantization > Encoding > Digital signal

- Sampling: conversion of a continuous-time signal to a discrete-time signal. The sampling frequency must satisfy $f_s \geq 2\ f_{max}$, where f_{max} is the maximum signal frequency.
- Quantization: mapping continuous amplitude values to a finite number of levels. This introduces quantization error/noise, which depends on the quantization step Δ.
- Encoding: assigning a binary word to each quantized level.



The overall process uses an ADC (Analog-to-Digital Converter). The reverse conversion uses a DAC (Digital-to-Analog Converter).

3. Characteristic Signal Parameters

Characteristic Signal Parameters

For a real continuous-time signal (CTS) $x(t)$ defined over the interval $[t_1, t_2]$, we define:

Mean Value:

$$x_{av} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} x(t)\, dt$$

RMS Value:

$$x_{rms} = \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} x^2(t)\, dt \right]^{1/2}$$

Instantaneous Power:

$$P(t) = x^2(t)$$

Average Power:

$$P_{av} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} x^2(t)\, dt$$

Energy:

$$E_x = \int_{t_1}^{t_2} x^2(t)\, dt$$

Notes on Energy and Power

- The energy of a signal corresponds to the positive area between its squared waveform (which is always nonnegative) and the horizontal time axis.
- Energy and power therefore cannot be negative. Squaring is essential, because positive and negative signal values could otherwise cancel in the integral.
- Squaring makes every signal value contribute positively to energy and power, regardless of its sign.
- For complex signals, the squared magnitude is used. It represents the actual signal magnitude and also produces a nonnegative quantity.

Energy and Power of Infinite-Duration Signals

For a generally complex signal, energy and power over the time interval $[-\infty, \infty]$ are given by:

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{E(T)}{2T}, \qquad \text{where } E_T = \int_{-T}^{T} |x(t)|^2 \, dt$$

4. Signal Classification by Energy and Power

Signal Classification by Energy and Power

A continuous-time signal $x(t)$ is classified as:

- Power **signal**: its average power P_{av} is finite and positive as the interval $[t_1, t_2]$ tends to infinity, i.e. $0 < P_{av} < \infty$
- Energy **signal**: its energy over the interval $[-\infty, \infty]$ is finite, i.e. $0 < E_x < \infty$.

For this to hold, the signal amplitude must decay to zero over time, i.e. as $|t| \to \infty$.

In some cases this does not occur. For example, the signal $x(t) = \cos(\omega_0 t)$, $-\infty < t < \infty$ oscillates indefinitely, while the signal $x(t) = t$ grows without bound as $t \to \infty$.

Notes on Energy and Power Signals

- A nonzero signal cannot be both an energy signal and a power signal:
$$0 < E_x < \infty \Rightarrow P_{av} = 0$$
- Energy signals have finite total energy and usually finite duration or an amplitude that decays to zero. Example: $x(t) = e^{-\alpha t}u(t),\ \alpha > 0$.
- Power signals have infinite energy but finite, positive average power. They are mainly used as ideal steady-state models.
- Periodic signals with bounded amplitude are power signals.
- A signal may belong to neither class when its energy and average power are infinite, or when the corresponding limits do not exist.
- Power signal applications include electrical networks, telecommunications carriers, audio, RF and wireless systems.
- The distinction between energy and power signals determines the appropriate method of signal analysis and processing.

5. Transformations of the Independent Variable

Transformations of the Independent Variable

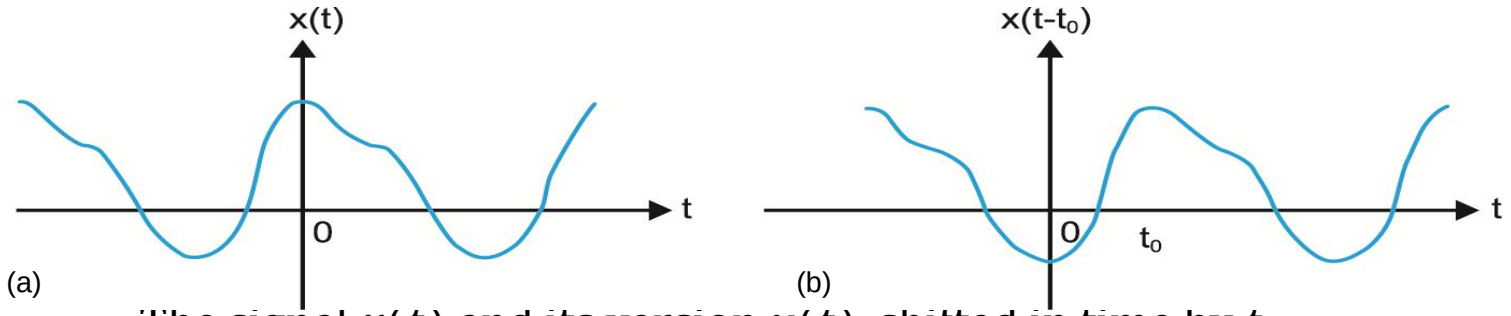
1. Time Shifting
2. Reflection (Time Reversal)
3. Time Scaling
4. Addition and Multiplication of Signals

Time Shifting of a Signal (1/2)

A signal $y(t)$ is a version of $x(t)$ shifted in time by t_0 when the independent variable t is replaced by $t - t_0$, i.e. when:

$$y(t) = x(t - t_0)$$

where t_0 is a real number representing the time shift.



The signal $x(t)$ and its version $y(t)$, shifted in time by t_0.

Time Shifting of a Signal (2/2)

- A time shift with positive $\boldsymbol{t_0}$ represents a **time delay** (a shift to the **right**). This commonly occurs when a signal is transmitted through a telecommunications system.
- A time shift with negative $\boldsymbol{t_0}$ represents a **time advance**, i.e. the signal shifts to the **left** along the time axis.
- For continuous-time signals, time advance is mainly theoretical. For discrete-time signals, it is implemented through storage and offline processing, using future samples to improve performance and develop more efficient signal processing algorithms.
- Time shifting does not change a signal's energy, i.e.:

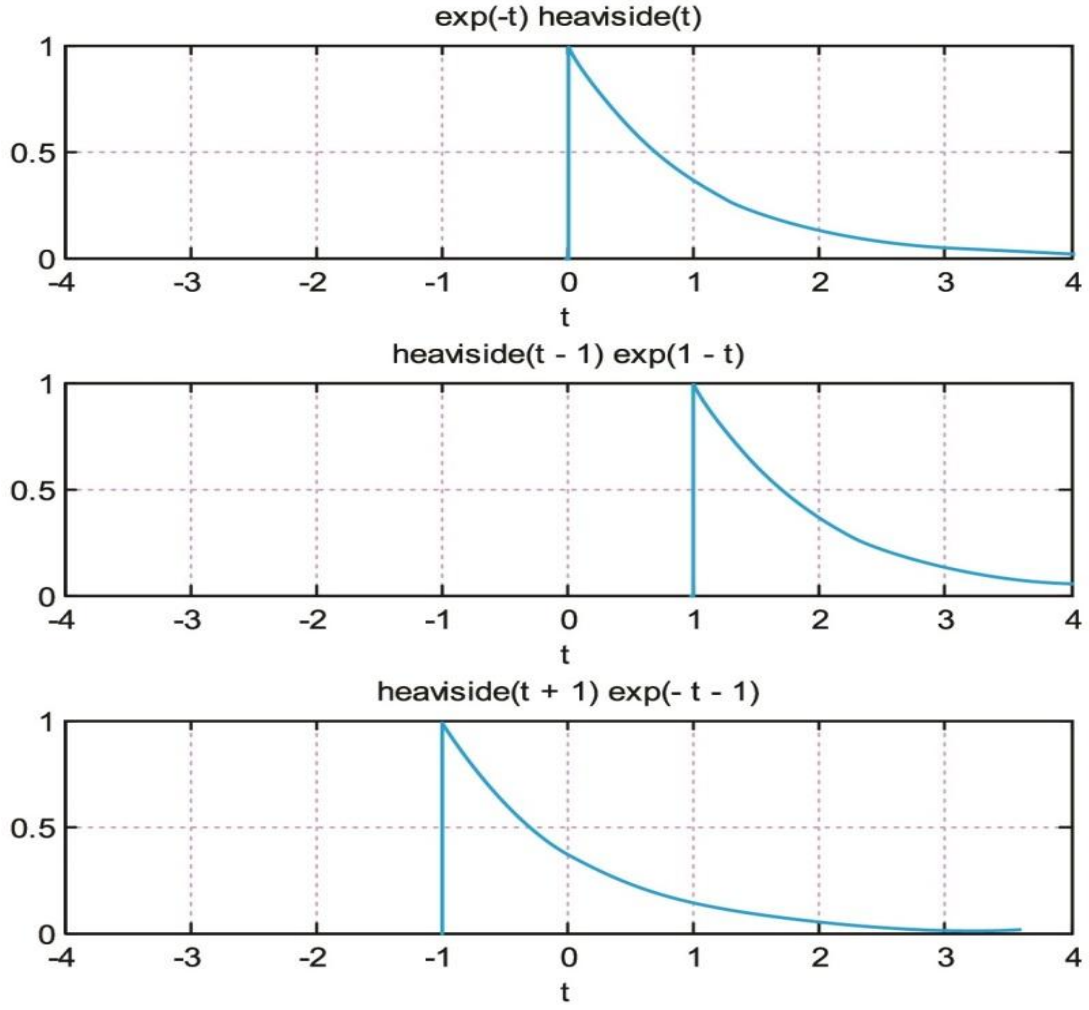
$$E[x(t)] = E[x(t - t_0)]$$

Example of Signal Time Shifting

(a) Original signal $x(t) = e^{-t}\,u(t)$

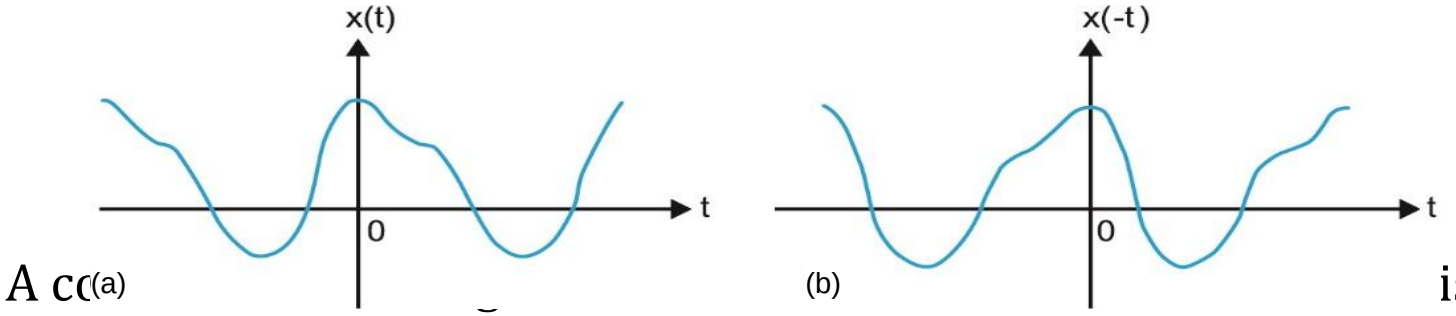
(b) Delay $y_1(t) = x(t-1)$

(c) Advance $y_2(t) = x(t+1)$



Signal Reflection

A signal $y(t)$ is the **reflection** of $x(t)$ about $t = 0$, i.e. about the vertical axis, when the independent variable t is replaced by $-t$, i.e. when $y(t) = x(-t)$.



A cc is

- Reflection interchanges the past and future of a signal.
- Reflection does not change a signal's energy, i.e.:

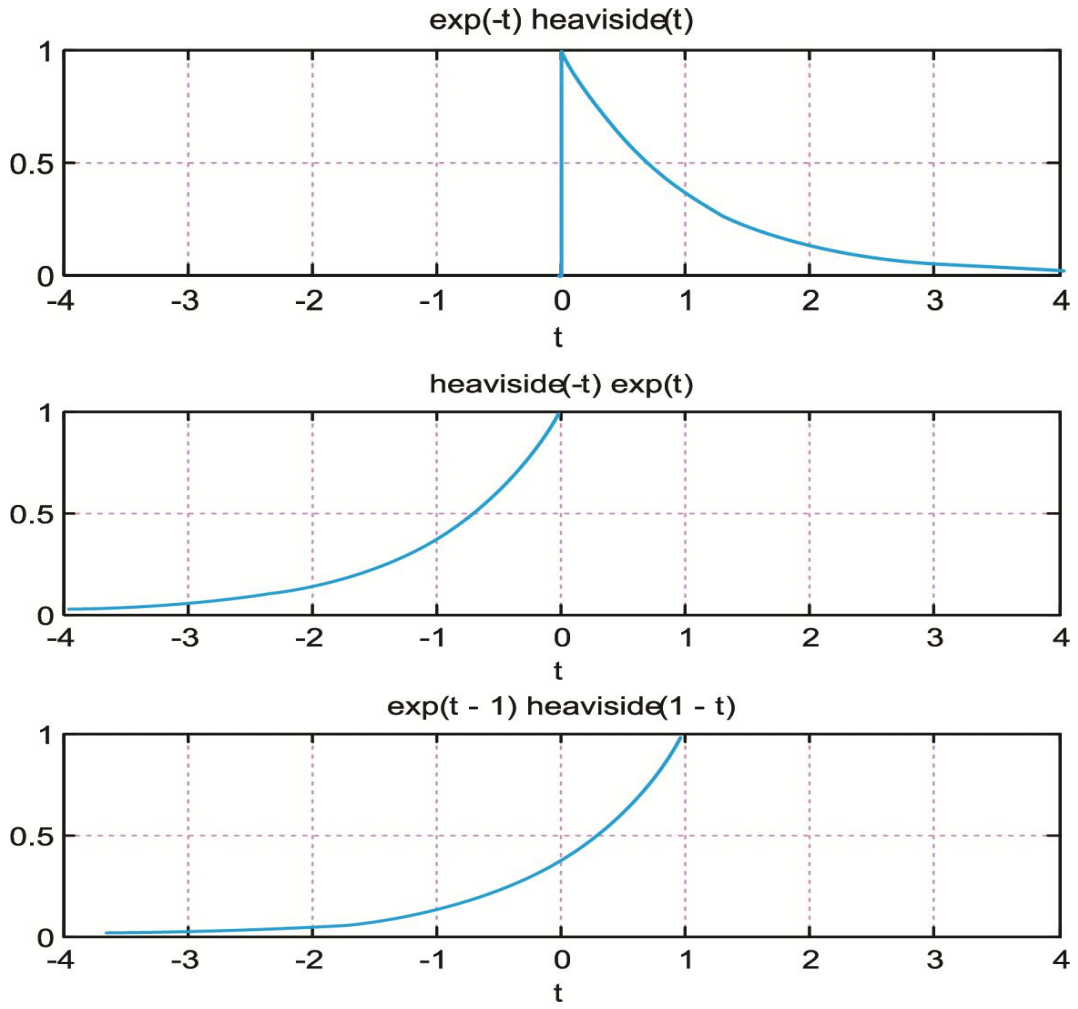
$$E[x(t)] = E[x(-t)]$$

Example of Signal Reflection

(a) Original signal $x(t) = e^{-t}u(t)$

(b) Reflection $y_1(t) = x(-t)$

(c) Reflection and shift
to the right $y_2(t) = x(1-t)$

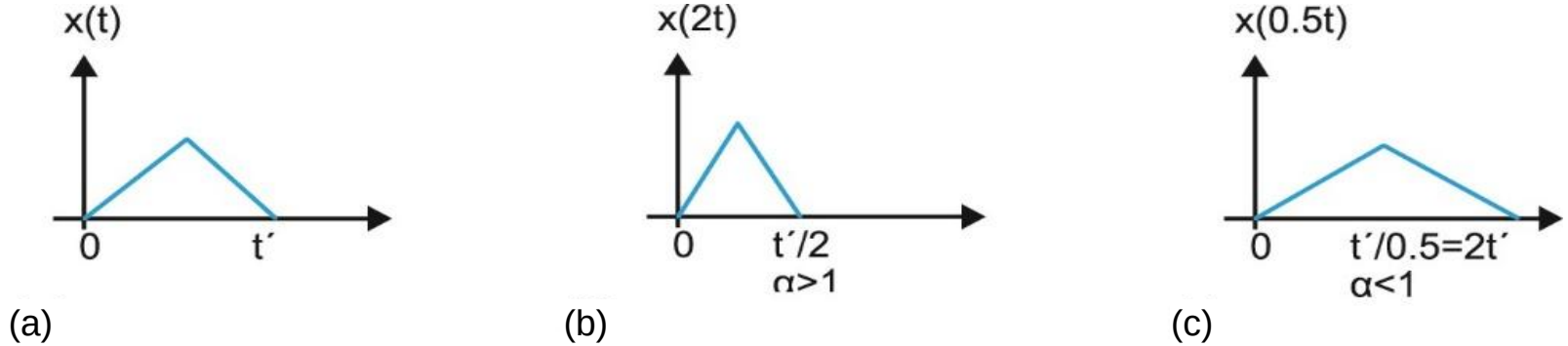


Time Scaling of a Signal

The signal $x_1(t)$ is a **time-compressed** version of $x(t)$ if $x_1(t) = x(at),\ a > 1$, i.e. if the independent variable t is replaced by αt.

The signal $x_2(t)$ is a **time-expanded** version of $x(t)$ if $x_2(t) = x(at),\ 0 < a < 1$

The origin $t = 0$ remains unchanged.



(a) Original signal, (b) time compression and (c) time expansion

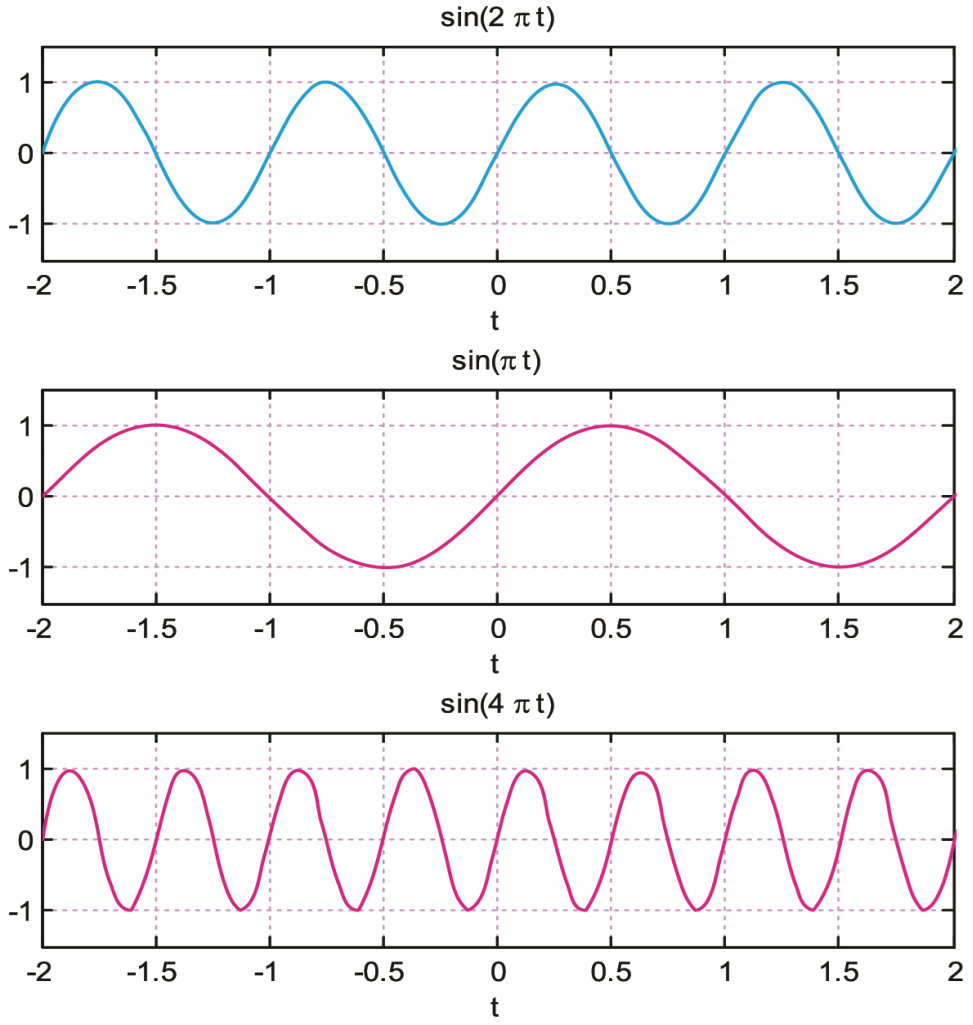
Scaling time by a constant α multiplies the signal energy by $\frac{1}{|\alpha|}$, i.e.: $\boldsymbol{E[y(t)] = \frac{1}{|a|} E[x(t)]}$

Example of Time Scaling

(a) Original signal $x(t) = \sin(2\pi t)$

(b) Expansion $y_1(t) = x(t/2)$

(c) Compression $y_2(t) = x(2t)$



Addition and Multiplication of Signals

Given two continuous-time signals $x_1(t)$ and $x_2(t)$, signal addition and multiplication are defined as the addition and multiplication of their corresponding instantaneous values, i.e.:

$$y(t) = x_1(t) + x_2(t)$$

$$z(t) = x_1(t) \cdot x_2(t)$$

- If the signals $x_1(t)$ and $x_2(t)$ do not overlap in time, the energy of the signal $y(t)$ equals the sum of their energies, i.e.:

$$E[y(t)] = E[x_1(t)] + E[x_2(t)]$$

6. Classes of Continuous-Time Signals

Classes of Continuous-Time Signals

1. Deterministic and Stochastic Signals
2. Causal and Noncausal Signals
3. Finite-Amplitude Signals
4. Finite-Duration and Infinite-Duration Signals
5. Even and Odd Signals
6. Periodic Signals

Deterministic and Stochastic Signals

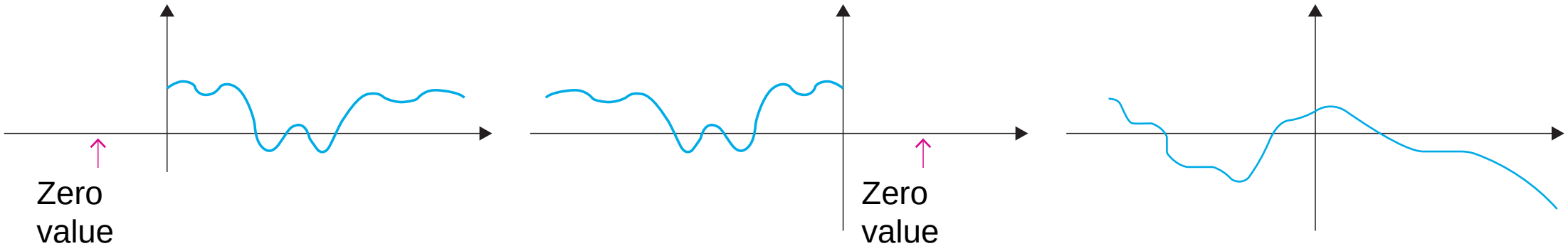
- A **deterministic** signal is one for which we can always write a mathematical expression that fully describes it at every instant, so its values can be determined in advance.
- A stochastic or random signal cannot be determined exactly in advance. We cannot write an analytical mathematical expression that describes it. **Noise** and many other real-world signals belong to this class.
- Probability theory is used to study stochastic signals.
- Stochastic signals are outside the scope of this course.

Signal Causality

A signal $x(t)$ is called:

- **causal** if $x(t) = 0$ for $t < 0$
- anticausal if $x(t) = 0$ for $t > 0$
- noncausal if $x(t) \neq 0$ for $t < 0$ and $t > 0$

Signals studied in practice are causal: they originate from physical systems and are generated after the system starts operating ($t = 0$).

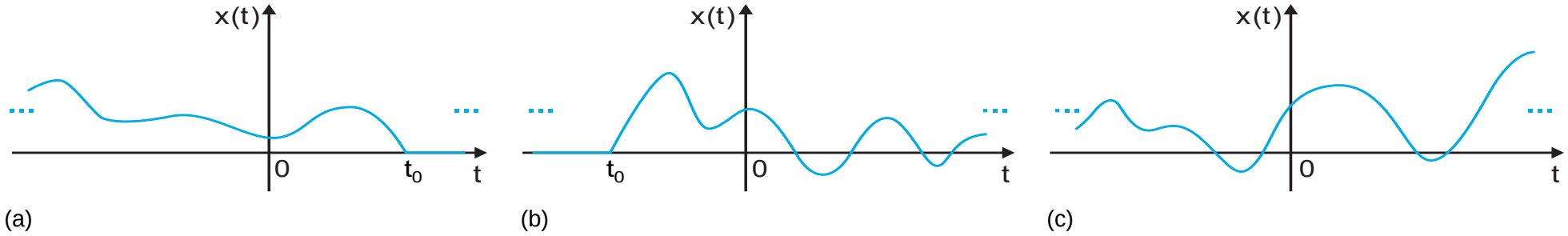


Examples: (a) causal, (b) anticausal and (c) noncausal signals

Signal Sidedness

An infinite-duration signal $x(t)$ is called:

- **Left-sided** if $x(t) = 0$ for $t > t_0$
- **Right-sided** if $x(t) = 0$ for $t < t_0$
- **Two-sided** if $x(t) \neq 0$ for $-\infty < t < \infty$



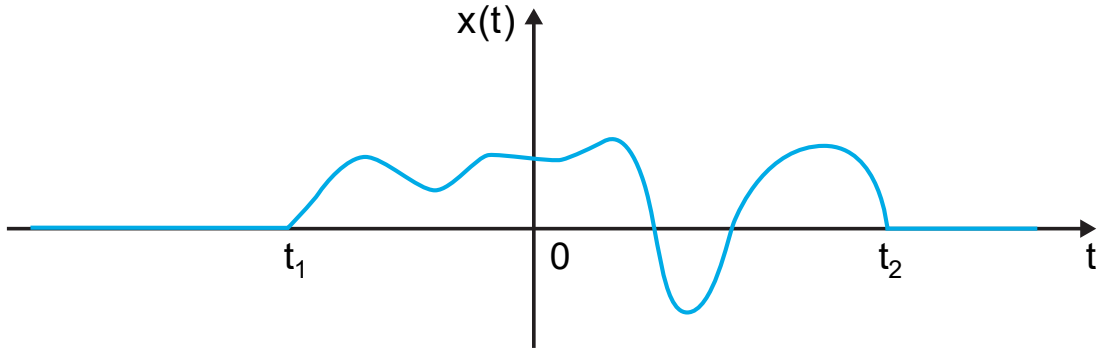
Signals: (a) left-sided, (b) right-sided and (c) two-sided

Signal sidedness is used to determine the region of convergence of the Laplace transform.

Signal Duration

A signal $x(t)$ has finite duration when $x(t) = 0, \quad t > t_1$ or $t < t_2$, where t_1 and t_2 are finite numbers $(t_1 < t_2)$.

If at least one of t_1 and t_2 tends to infinity, the signal has infinite duration.



Finite-duration signal

Finite-Amplitude Signals

A signal $x(t)$ has **finite amplitude** if:

$$\exists M < \infty: \quad |x(t)| < M, \qquad \forall t$$

Equivalently, we can use the term bounded-amplitude signal.

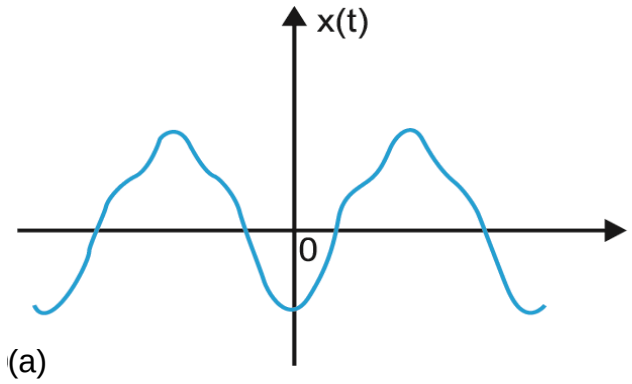
Even and Odd Signals

A signal $x(t)$ is **even** and exhibits **even symmetry** if:

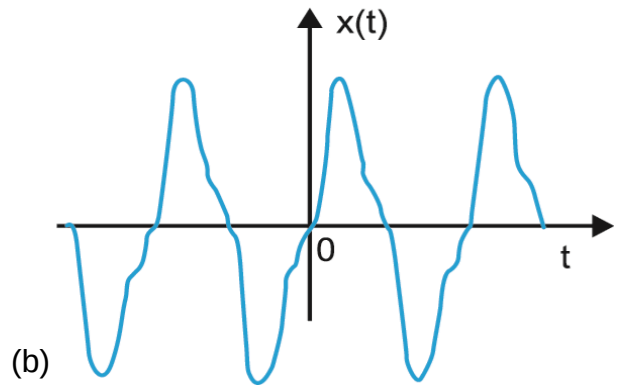
$$x(-t) = x(t) \qquad -\infty < t < +\infty$$

A signal $x(t)$ is **odd** and exhibits **odd symmetry** if:

$$x(-t) = -x(t) \qquad -\infty < t < +\infty$$



(a) Even signal



(b) Odd signal

Even and Odd Signals

Every signal can be expressed as the **sum of an even signal $x_e(t)$** and an odd signal $x_o(t)$, i.e.:

$$x(t) = x_e(t) + x_o(t)$$

where $x_e(t)$ and $x_o(t)$ are given by:

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

It is easy to show that:

- The product of two **even** signals or two odd signals is an even signal.
- The product of an even signal and an **odd** signal is an odd signal.

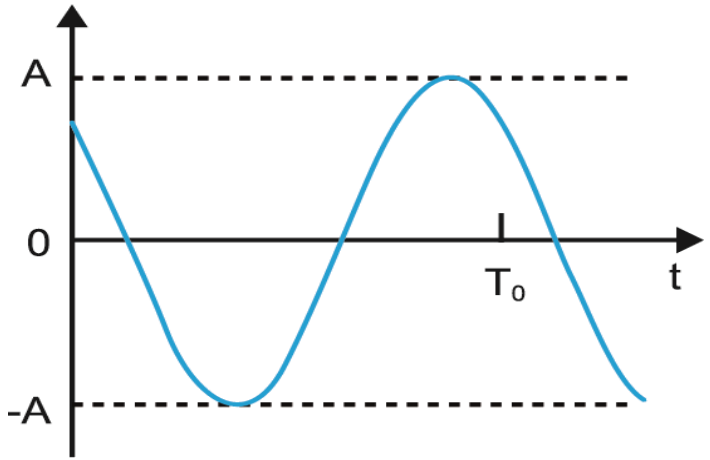
Decomposition into even and odd components is especially useful in signal analysis. Time symmetries simplify calculations and help explain how a signal behaves under transformations.

Signal Periodicity (1/2)

- A continuous-time signal $x(t)$ is **periodic** if there is a positive number T such that $x(t) = x(t + mT)$ for every time instant t and integer m.
- The requirement for every time instant ***t*** means that the signal has infinite duration, i.e. it is defined for $-\infty < t < \infty$.
- The quantity T is the **period**, measured in seconds. The smallest period is the **fundamental period** (T_0). It is the shortest time interval after which the periodic signal repeats.
- The quantity $f_0 = 1/T_0$ is the fundamental frequency. It gives the number of repetitions per second and is measured in Hertz (Hz).
- The quantity $\Omega_0 = 2\pi/T_0 = 2\pi f_0$ is the **angular (analog) frequency** and is measured in rad/sec.

Signal Periodicity (2/2)

Classic examples of periodic signals are **sine** and cosine signals, described by an expression of the form $x(t) = A\,cos\,(\Omega_0 t + \theta)$, where A is the **amplitude** and θ is the **phase angle**, or simply the **phase.**



The sinusoidal signal $x(t) = A\,cos(\Omega_0 t + \pi/4)$

Notes on Signal Periodicity

- The requirement of infinite duration means that periodic signals cannot be realized exactly in practical applications. They are, however, very useful steady-state models and form the basis of Fourier analysis.
- Periodic signals are analyzed using Fourier series, which express the signal as a sum of harmonics with frequencies $k\Omega_0$.
- Aperiodic signals are described by the Fourier transform. It can also be viewed as the limit of the Fourier series as the fundamental period T_0 tends to infinity: the discrete spectrum becomes increasingly dense and then continuous.
- The response of linear systems to sinusoidal inputs is fundamental to the study of continuous-time systems.
- A cosine signal is a sine signal with a phase shift of $\theta = \pm\pi/2$. We therefore often refer to both collectively as sinusoidal signals.

Notes on Signal Periodicity

- Sinusoidal signals occur frequently in nature and technology, e.g. alternating currents, telecommunications carriers and acoustic oscillations.
- Through Fourier analysis, even nonsinusoidal signals can be represented as a sum or integral of sinusoidal components.
- An important property is that derivatives and integrals of sinusoidal signals remain sinusoidal, with changes in amplitude and phase.
- A constant signal of the form $x(t) = A$ can be regarded as periodic with an unspecified period, since it can be written as $x(t) = A = A\cos(0t)$ .. In this case there is no fundamental period.

Periodicity of a Sum of Periodic Signals

Let $x(t)$ and $y(t)$ be periodic signals with fundamental periods T_1 and T_2, respectively. We examine the conditions under which their **sum** $z(t) = x(t) + y(t)$ is periodic.

- Since $x(t)$ and $y(t)$ are periodic with fundamental periods T_1 and T_2, respectively:

$$x(t) = x(t + mT_1), \qquad m = 1,2,\ldots.$$
$$y(t) = y(t + nT_2), \qquad n = 1,2,\ldots.$$

- If the sum $z(t) = x(t) + y(t)$ is periodic with **fundamental period $\boldsymbol{T}$**, then $z(t) = z(t + T)$, which can be written as:

$$z(t + T) = x(t + mT_1) + y(t + nT_2)$$

- This relation is satisfied only if:

$$T = mT_1 = nT_2 \Rightarrow \frac{\boldsymbol{T_1}}{\boldsymbol{T_2}} = \frac{\boldsymbol{n}}{\boldsymbol{m}}$$

This is the necessary condition for the sum of two periodic signals to be periodic.

Exercises

Exercise 1

A signal $x(t)$ enters a system whose output is the signal $y(t) = a\,x(bt)$. If $x(t)$ has energy E_x over the interval $(-\infty, +\infty)$, calculate the energy E_y of $y(t)$ over the same interval.

Answer: The energy E_x of the signal $x(t)$ over the interval $(-\infty, +\infty)$ is given by:

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt$$

Similarly, the energy E_y of the signal $y(t)$ is:

$$E_y = \int_{-\infty}^{+\infty} |y(t)|^2 dt = a^2 \int_{-\infty}^{+\infty} |x(bt)|^2 dt$$

We evaluate the integral by setting $\lambda = bt$, which gives:

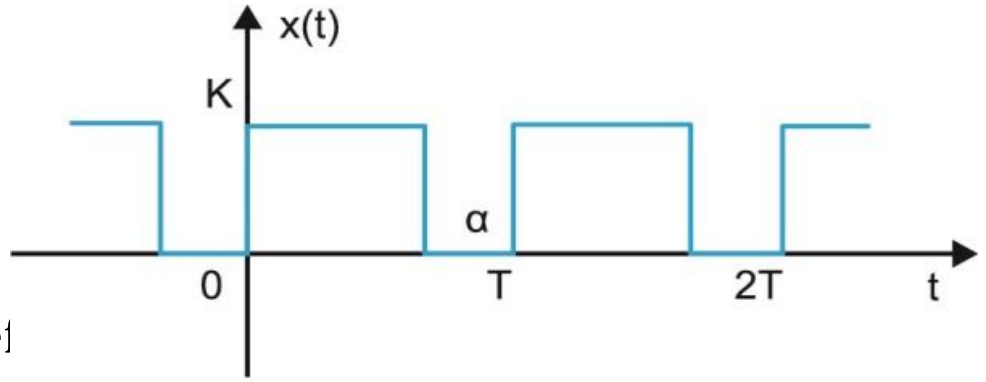
$$E_y = \frac{a^2}{b} \int_{-\infty}^{+\infty} x^2(\lambda)\, d\lambda$$

Therefore: $E_y = \frac{a^2}{b} E_x$

Exercise 2

Calculate the mean value of the signal $x(t)$ as a function of α and K.

Answer: The graph shows that
the signal $x(t)$ is periodic
with period T and amplitude K.



Its mean value over the interval $[t_1, t_2] = [0, \mathrm{T}]$ is there

$$\bar{x}(t) = x_{av} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} x(t)dt = \frac{1}{T}\int_0^T x(t)dt =$$

$$= \frac{1}{T}\int_0^{T-a} K\,dt = \left[\frac{K}{T}\,t\right]_0^{T-a} = K\left(1 - \frac{a}{T}\right)$$

The mean value of a periodic signal is its DC component.
The mean alone does not fully describe the signal's behavior.

Exercise 3

Calculate the energy of the signal $x(t) = e^{-|t|}$.

Answer: From the equation defining the signal, we obtain:

$$|x(t)|^2 = e^{-2|t|} = \begin{cases} e^{+2t} & \text{for } t < 0 \\ e^{-2t} & \text{for } t > 0 \end{cases}$$

Therefore, the signal energy is given by:

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{0} e^{2t} dt + \int_{0}^{\infty} e^{-2t} dt =$$

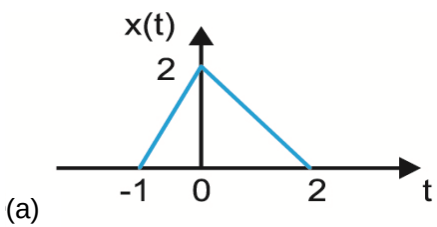
$$= \left[\frac{1}{2} e^{2t}\right]_{-\infty}^{0} + \left[-\frac{1}{2} e^{2t}\right]_{0}^{\infty} = \frac{1}{2} + \frac{1}{2} = 1$$

Since the signal energy is finite, $x(t)$ is an **energy signal.**

Exercise 4

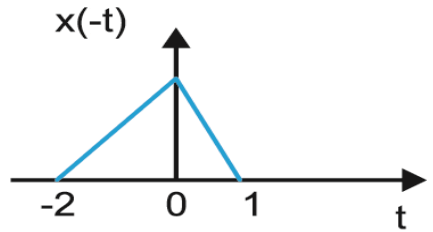
A signal $x(t)$ is defined by $x(t) = \begin{cases} 2t+2, & -1 \le t < 0 \\ 2-t, & 0 \le t < 2 \\ 0, & \text{otherwise} \end{cases}$. Plot the signals $y_1(t) = x(-t)$, $y_2(t) = x(t+1)$, $y_3(t) = x(2t)$, $y_4(t) = x(t/2)$, $y_5(t) = x(1-t)$, $y_6(t) = x(2t-2)$.

Answer: The original signal x(t) is shown in graph (a):



(a)

The signal $y_1(t)$ is the reflection of $x(t)$ and is obtained by replacing t with $-t$:



(b)

$$y_1(t) = \begin{cases} 2(-t)+2, & -1 \le -t < 0 \\ 2-(-t), & 0 \le -t < 2 \\ 0, & \text{otherwise} \end{cases} = \begin{cases} -2t+2, & 1 \ge t > 0 \\ 2+t, & 0 \ge t > -2 \\ 0, & \text{otherwise} \end{cases} = \begin{cases} t+2, & -2 \le t < 0 \\ 2-2t, & 0 \le t < 1 \\ 0, & \text{otherwise} \end{cases}$$

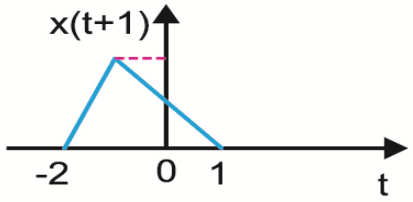
The graph of the signal $y_1(t) = x(-t)$ is shown in (b).

Exercise 4 (continued)

The signal $y_2(t)$ is a time-shifted version of $x(t)$.

$$y_2(t) = \begin{cases} 4 + 2t, & -2 \le t < -1 \\ 1 - t, & -1 \le t < 1 \\ 0, & \text{otherwise} \end{cases}$$

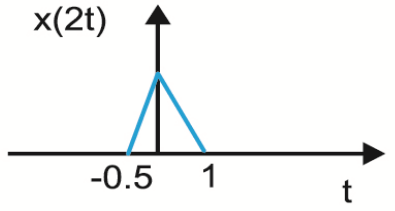
(c)



The signals $y_3(t)$ and $y_4(t)$ are time-compressed and time-expanded versions of $x(t)$, respectively:

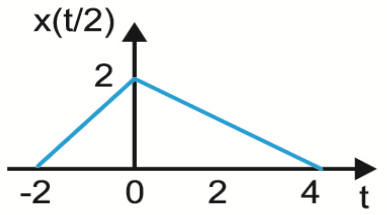
$$y_3(t) = \begin{cases} 2 + 4t, & -\frac{1}{2} \le t < 0 \\ 2 - 2t, & 0 \le t < 1 \\ 0, & \text{otherwise} \end{cases}$$

(d)



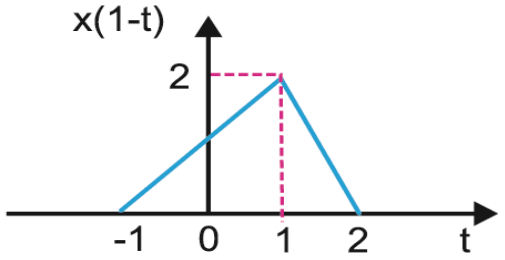
$$y_4(t) = \begin{cases} 2 + t, & -2 \le t < 0 \\ 2 - \frac{t}{2}, & 0 \le t < 4 \\ 0, & \text{otherwise} \end{cases}$$

(e)



Exercise 4 (continued)

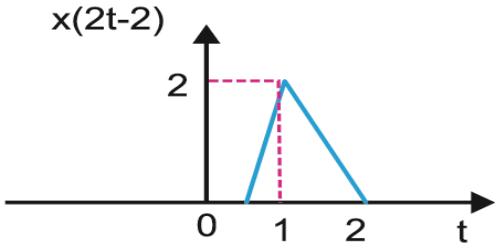
The signal $y_5(t)$ is a version of $y_1(t)$ shifted to the right, i.e. a shifted reflection of $x(t)$.



(f)

The signal $y_6(t) = x[2(t-1)]$ is an expanded version of $x(t-1)$, i.e. the time-shifted version of $x(t)$.

(g)



Note: transformations do not commute, so their order affects the result. Specifically, $x(at-b) \neq x(a(t-b))$. The correct approach is to rewrite the function, e.g. by factoring, in a form that reveals the order of operations.

Exercise 5

Calculate and plot the even and odd components of the signal:

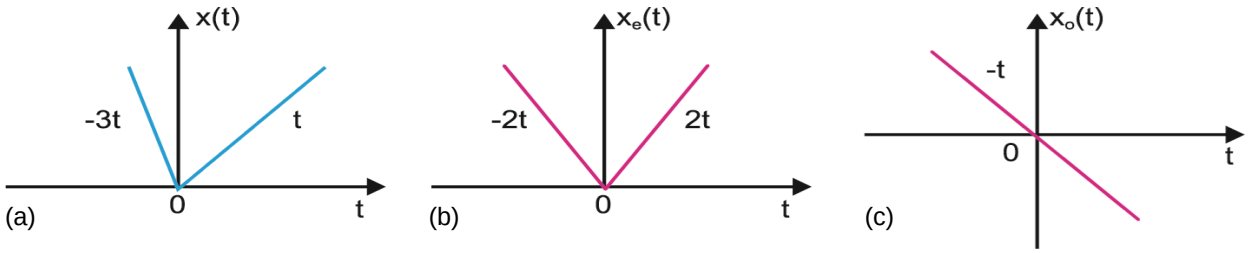
$$x(t) = \begin{cases} -3t, & t < 0 \\ t, & t \geq 0 \end{cases}$$

Answer: The even signal $x_e(t)$ (the even component of $x(t)$) is:

$$x_e(t) = 0.5\,[x(t) + x(-t)] = \begin{cases} 0.5\,[-3t - t], & t < 0 \\ 0.5\,[t + 3t], & t \geq 0 \end{cases} = \begin{cases} -2t, & t < 0 \\ 2t, & t \geq 0 \end{cases}$$

while the odd signal $x_o(t)$ (the odd component of $x(t)$) is:

$$x_o(t) = 0.5\,[x(t) - x(-t)] = \begin{cases} 0.5\,[-3t + t], & t < 0 \\ 0.5\,[t - 3t], & t \geq 0 \end{cases} = \begin{cases} -t, & t < 0 \\ t, & t \geq 0 \end{cases}$$



Graphs: (a) $x(t)$, (b) $x_e(t)$ and (c) $x_o(t)$. We observe that $x(t) = x_e(t) + x_o(t)$

Exercise 6

Determine the even and odd components of the signal $x(t) = e^{j\Omega t}$.

Answer: Using the definitions of the components $x_e(t)$ and $x_o(t)$ of the signal $x(t) = e^{j\Omega t}$, together with Euler's formulas:

$$e^{+j\Omega t} = \cos(\Omega t) + j\sin(\Omega t) \quad \text{and} \quad e^{-j\Omega t} = \cos(\Omega t) - j\sin(\Omega t)$$

we obtain:

$$x_e(t) = 0.5\,[x(t) + x(-t)] = 0.5\left[e^{j\Omega t} + e^{-j\Omega t}\right]$$
$$= 0.5\,[\cos(\Omega t) + j\sin(\Omega t) + \cos(\Omega t) - j\sin(\Omega t)\,]$$
$$= \cos(\Omega t)$$

and:

$$x_o(t) = 0.5\,[x(t) - x(-t)] = 0.5\left[e^{j\Omega t} - e^{-j\Omega t}\right]$$
$$= 0.5\,[\cos(\Omega t) + j\sin(\Omega t) - \cos(\Omega t) + j\sin(\Omega t)\,]$$
$$= j\sin(\Omega t)$$

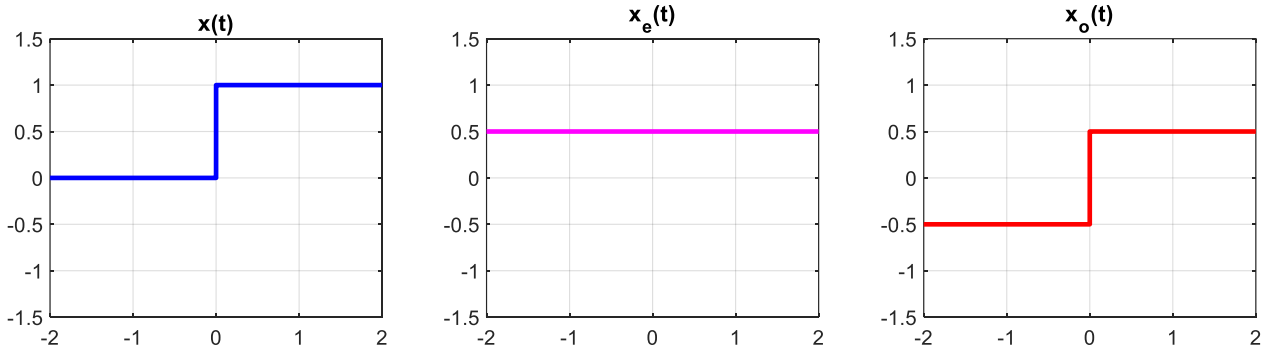
Exercise 7

Determine the even and odd components of the unit step function $u(t)$.

Answer: From the definitions of the even and odd components, we obtain:

$$u_e(t) = 0.5\,[u(t) + u(-t)] = \begin{cases} 0.5\,[0+1], & t < 0 \\ 0.5\,[1+0], & t > 0 \end{cases} = 0.5$$

$$u_o(t) = 0.5\,[u(t) - u(-t)] = \begin{cases} 0.5\,[0-1], & t < 0 \\ 0.5\,[1-0], & t > 0 \end{cases} = \begin{cases} -0.5, & t < 0 \\ 0.5, & t > 0 \end{cases}$$



(a) Unit step function, (b) even component, (c) odd component

Exercise 8

Show that the signal $x(t) = A\,cos(\Omega_0 t + \theta)\,, -\infty < t < \infty$ is periodic and find its fundamental period.

Answer: For the signal $x(t)$ to be periodic, there must be a positive number T such that $x(t+T) = x(t)$. Using the equation defining the signal, this relation becomes:

$$x(t+T) = A\,cos(\Omega_0(t+T) + \theta) = A\,cos(\Omega_0 t + \Omega_0 T + \theta)$$

The cosine function is periodic with period $T = 2\pi m$. For $x(t+T) = x(t)$, or $A\,cos(\Omega_0 t + \Omega_0 T + \theta) = A\,cos(\Omega_0 t + \theta)$, to hold, we require $\Omega_0 T = 2\pi m$. Therefore $T = 2\pi m/\Omega_0, \Omega_0 \neq 0$. The fundamental period is $T_0 = 2\pi/\Omega_0$.

Exercise 9

Determine whether each of the following signals is periodic.

(a) $y(t) = 3sin(3\pi t) + 2sin(4\pi t)$

(b) $y(t) = 4cos(100t) + 5sin(70\sqrt{2}\, t)$

(c) $y(t) = \sqrt{2}\, cos(12t) + 1.5\, cos(2\pi t)$

Answer: (a) The angular frequencies of the signal components are $\Omega_1 = 3\pi$ and $\Omega_2 = 4\pi$. Since the ratio $\Omega_2/\Omega_1 = 4/3$ is rational, the signal $y(t)$ is periodic.

(b) We have $\Omega_1 = 100$ and $\Omega_2 = 70\sqrt{2}\,(\Omega_2/\Omega_1) = 70\sqrt{2}\,\frac{1}{100} = 7\sqrt{2}\,\frac{1}{10}$. Therefore $y(t)$ is not periodic.

(c) We have $\Omega_1 = 12$ and $\Omega_2 = 2\pi(\Omega_2/\Omega_1) = \pi/6$. Therefore the signal $y(t)$ is not periodic.

Exercise 10

Calculate the fundamental period of each of the following periodic signals:

(a) $x_1(t) = e^{j\,0.25\,\pi t}$

(b) $x_2(t) = cos(0.2\,\pi t)$

(c) $x_3(t) = 2\,cos(0.1\,\pi t) + \ 2\,sin(0.2\,\pi t)$

Answer: (a) The signal $x_1(t)$ can be written as $e^{\,j2\pi t/8}$. Since $2\pi/T_0 = 2\pi/8$,, the fundamental period is $T_0 = 8$.

(b) The fundamental period of $x_2(t) = cos(0.2\,\pi t) = \cos\left(\frac{2\pi}{10}t\right)$ is $T_0 = 10$.

(c) The fundamental period of $2cos(0.1\,\pi t)$ is $T_0 = \frac{2\pi}{0.1\pi} = 20$,, while that of $2sin(0.2\pi t)$ is

$T_0 = \frac{2\pi}{0.2\pi} = 10$. Since 20 is an integer multiple of 10, it is also a period of $2\,sin(0.2\,\pi t)$.
Therefore, the fundamental period of $x_3(t)$ is $T_0 = 20$.

Exercise 11

Find the fundamental period of the signal $x(t) = cos(7\pi t) + 3\,sin(8\pi t)$.

Answer: If the signal $x(t)$ is periodic with period T, then $x(t) = x(t+T)$.. The signal $x(t+T)$ is defined as:

$$x(t+T) = cos[7\pi(t+T)] + 3\,sin[8\pi(t+T)]$$
$$= cos(7\pi t + 15\pi T) + 3\,sin(8\pi t + 8\pi T)$$

Since the periods of the two sinusoidal signals have the form $T_1 = 2\pi m$ and $T_2 = 2\pi n$, the periodicity condition is satisfied if:

$$7\pi T = 2\pi m \text{ and } 8\pi T = 2\pi n$$

Solving for T, we obtain:

$$T = \frac{2m}{7} = \frac{2n}{8} \text{ or equivalently } \frac{n}{m} = \frac{8}{7}$$

The two numbers above have no common divisor other than 1.

Hence $n = 8$ and $m = 7$,, which gives $T = 2n/8 = 2$

Exercise 12

Determine whether the following signal is periodic. If so, calculate its fundamental period.

$$x(t) = \cos(2t)\cos\left(\frac{3}{2}t\right) + \sin^2\left(\frac{5}{4}t\right)$$

Answer: To find the period, we express the product of sinusoidal functions as a sum, using the identity:

$$\cos A\,\cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

which gives:

$$\cos(2t)\cos\left(\frac{3}{2}t\right) = \frac{1}{2}\left[\cos\left(\frac{1}{2}t\right) + \cos\left(\frac{7}{2}t\right)\right]$$

Exercise 12 (continued)

We know that:

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

therefore:

$$\sin^2\left(\frac{5}{4}t\right) = \frac{1}{2}\left(1 - \cos\left(\frac{5}{2}t\right)\right)$$

Hence, the signal $x(t)$ is:

$$x(t) = \frac{1}{2}\cos\left(\frac{1}{2}t\right) + \frac{1}{2}\cos\left(\frac{7}{2}t\right) + \frac{1}{2} - \frac{1}{2}\cos\left(\frac{5}{2}t\right)$$

Exercise 12 (continued)

The given signal is therefore the sum of a constant (1/2) and three cosine functions with angular frequencies:

$$\Omega_1 = \frac{1}{2}, \qquad \Omega_2 = \frac{7}{2}, \qquad \Omega_3 = \frac{5}{2}$$

The corresponding periods are:

$$T_1 = 4\pi, \qquad T_2 = \frac{4\pi}{7}, \qquad T_3 = \frac{4\pi}{5}$$

The signal is periodic if the periods have a common multiple, or equivalently if the angular frequency ratios are rational:

$$\frac{\Omega_2}{\Omega_1} = 7, \qquad \frac{\Omega_3}{\Omega_1} = 5$$

therefore the signal is periodic.

Exercise 12 (continued)

We seek the smallest period $T_0 > 0$ such that:

$$\frac{1}{2}T_0 = 2\pi k_1, \qquad \frac{7}{2}T_0 = 2\pi k_2, \qquad \frac{5}{2}T_0 = 2\pi k_3$$

The first equation gives:

$$T_0 = 4\pi k_1$$

For $k_1 = 1$, we obtain:

$$\frac{7}{2} \cdot 4\pi = 14\pi = 2\pi \cdot 7, \qquad \frac{5}{2} \cdot 4\pi = 10\pi = 2\pi \cdot 5$$

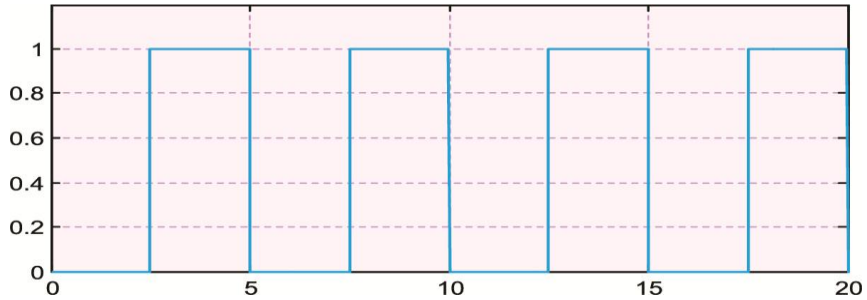
Thus $T_0 = 4\pi$ satisfies all components and is the smallest such value. The fundamental period is therefore:

$$T_0 = 4\pi$$

Examples of Periodic Signals

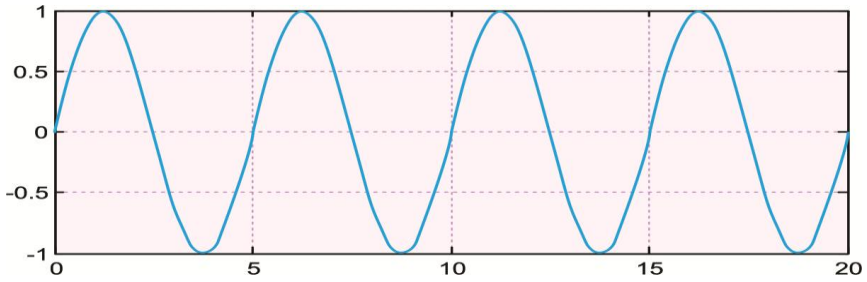
```
[y,t] = gensig('square',5,20,0.01);

plot(t,y); ylim([0, 1.2]); grid on
```




```
[y,t] = gensig('sin',5,20,0.01);

plot(t, y); ylim([-1, 1]); grid on
```



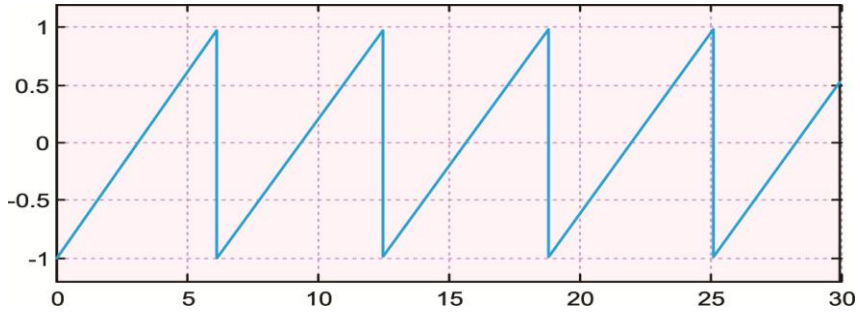

```
[y,t] = gensig('pulse',5,20,0.01);

plot(t, y); ylim([0, 1.2]); grid on
```

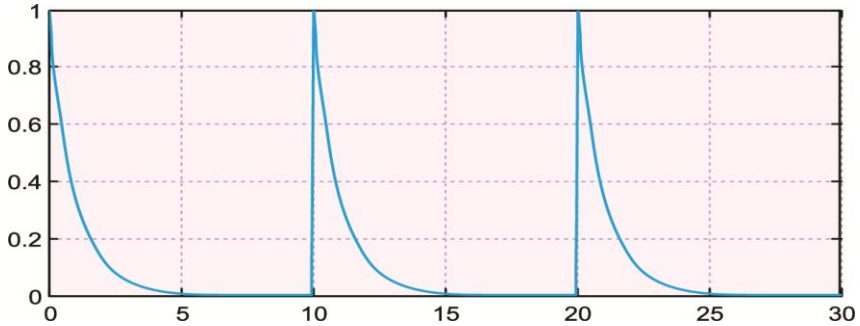


Examples of Periodic Signals

```
t = 0 : 0.1: 30;
y = sawtooth(t); plot(t, y); ylim([-1.2, 1.2]);
grid on
```




```
N = 3;
t = 0 : 0.1 : 10;
x = exp(t); y = repmat(x, 1, N);
ty = linspace(0,10*N,length(y)); plot(ty, y); grid on
```




```
N = 3;
x = [1 2 4 -2 1 0];
y = repmat(x, 1, N); ty = linspace(0,10*N,length(y));
plot(ty, y); grid on
```

