Δεύτερη Σειρα ασκήσεων:

1) Βιβλίο Βαρουχάκη Γ' Λυκείου, Κεφ. 2:

Ασκήσεις 16, 18, 21, 23, 26, 36

2)

Test the following series for convergence or divergence and give a reason for your decision in each case.

1. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$.
2. $\sum_{n=1}^{\infty} \frac{(n!)^2}{2^{n^2}}$.
3. $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$.
4. $\sum_{n=1}^{\infty} \frac{3^n n!}{n^n}$.
5. $\sum_{n=1}^{\infty} \frac{n!}{3^n}$.
6. $\sum_{n=1}^{\infty} \frac{n!}{2^{2n}}$.
7. $\sum_{n=2}^{\infty} \frac{1}{(\log n)^{1/n}}$
8. $\sum_{n=1}^{\infty} (n^{1/n} - 1)^n$.
9. $\sum_{n=1}^{\infty} e^{-n^2}$.
10. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - e^{-n^2}\right)$.
11. $\sum_{n=1}^{\infty} \frac{(1000)^n}{n!}$.
12. $\sum_{n=1}^{\infty} \frac{n^{n+1/n}}{(n + 1/n)^n}$.

3)

Each of the series in Exercises 1 through 10 is a telescoping series, or a geometric series, or some related series whose partial sums may be simplified. In each case, prove that the series converges and has the sum indicated.

1. $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{2}$.
2. $\sum_{n=1}^{\infty} \frac{2}{3^{n-1}} = 3$.
3. $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1} = \frac{3}{4}$.
4. $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{6^n} = \frac{3}{2}$.
5. $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n^2 + n}} = 1$.
6. $\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)(n+3)} = \frac{1}{4}$.
7. $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} = 1$.
8. $\sum_{n=1}^{\infty} \frac{2^n + n^2 + n}{2^{n+1} n(n+1)} = 1$.
9. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2n+1)}{n(n+1)} = 1$.
10. $\sum_{n=2}^{\infty} \frac{\log[(1 + 1/n)^n (1 + n)]}{(\log n^n)[\log (n+1)^{n+1}]} = \log_2 \sqrt{e}$.