Τυπολόγιο

Κυλινδρικές σε Καρτεσιανές

Αλλαγές μεταβλητών:
$$\begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \\ z = z \end{cases}$$

Αλλαγές συνιστωσών:
$$\begin{cases} A_x = A_\rho \dfrac{x}{\sqrt{x^2+y^2}} - A_\phi \dfrac{y}{\sqrt{x^2+y^2}} \\ A_y = A_\rho \dfrac{y}{\sqrt{x^2+y^2}} + A_\phi \dfrac{x}{\sqrt{x^2+y^2}} \\ A_z = A_z \end{cases}$$

Καρτεσιανές σε Κυλινδρικές

Αλλαγές μεταβλητών:
$$\begin{cases} \rho = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \quad \begin{cases} \sin\phi = \dfrac{y}{\sqrt{x^2+y^2}} \\ \cos\phi = \dfrac{x}{\sqrt{x^2+y^2}} \end{cases}$$

Αλλαγές συνιστωσών:
$$\begin{cases} A_\rho = A_x\cos\phi + A_y\sin\phi \\ A_\phi = -A_x\sin\phi + A_y\cos\phi \\ A_z = A_z \end{cases}$$

Σφαιρικές σε Καρτεσιανές

Αλλαγές μεταβλητών:
$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$

Αλλαγές συνιστωσών:
$$\begin{cases} A_x = \dfrac{A_r x}{\sqrt{x^2+y^2+z^2}} + \dfrac{A_\theta xz}{\sqrt{(x^2+y^2)(x^2+y^2+z^2)}} - \dfrac{A_\phi y}{\sqrt{x^2+y^2}} \\ A_y = \dfrac{A_r y}{\sqrt{x^2+y^2+z^2}} + \dfrac{A_\theta yz}{\sqrt{(x^2+y^2)(x^2+y^2+z^2)}} + \dfrac{A_\phi x}{\sqrt{x^2+y^2}} \\ A_z = \dfrac{A_r z}{\sqrt{x^2+y^2+z^2}} - \dfrac{A_\theta\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}} \end{cases}$$

Καρτεσιανές σε Σφαιρικές

Αλλαγές μεταβλητών:
$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \cos^{-1}\dfrac{z}{\sqrt{x^2+y^2+z^2}} \\ \phi = \tan^{-1}(y/x) \end{cases} \quad \begin{cases} \cos\theta = \dfrac{z}{\sqrt{x^2+y^2+z^2}} \\ \sin\theta = \dfrac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}} \\ \cos\phi = \dfrac{x}{\sqrt{x^2+y^2}} \\ \sin\phi = \dfrac{y}{\sqrt{x^2+y^2}} \end{cases}$$

Αλλαγές συνιστωσών:
$$\begin{cases} A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta \\ A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta \\ A_\phi = -A_x\sin\phi + A_y\cos\phi \end{cases}$$

Εσωτερικά γινόμενα

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta_{AB}$
$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$
$\mathbf{A}\cdot\mathbf{A} = A^2$
$\mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B}+\mathbf{A}\cdot\mathbf{C}$
$\nabla\cdot(f\mathbf{A}) = (\nabla\cdot\mathbf{A})f + \mathbf{A}\cdot(\nabla f)$

Εξωτερικά γινόμενα

$\mathbf{A}\times\mathbf{B} = AB\sin\theta_{AB}\ \hat{\mathbf{n}}$
$\mathbf{A}\times\mathbf{B} = -\mathbf{B}\times\mathbf{A}$
$\mathbf{A}\times\mathbf{A} = 0$
$\mathbf{A}\times(\mathbf{B}+\mathbf{C}) = \mathbf{A}\times\mathbf{B}+\mathbf{A}\times\mathbf{C}$
$\nabla\times(f\mathbf{A}) = (\nabla\times\mathbf{A})f + (\nabla f)\times\mathbf{A}$

Τριπλά γινόμενα

$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$
$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$
$\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

Διάφορα Θεωρήματα

Stoke's $\quad \oint_L \mathbf{A}\cdot d\mathbf{l} = \iint_S (\nabla\times\mathbf{A})\cdot d\mathbf{S}$

Απόκλισης $\quad \oiint_S \mathbf{A}\cdot d\mathbf{S} = \iiint_V (\nabla\cdot\mathbf{A})\ dV$

Καρτεσιανές: (x,y,z)
Βαθμωτό πεδίο: $f(x,y,z)$
Διανυσματικό πεδίο:

$$\begin{aligned}\mathbf{A}(x,y,z) &= A_x(x,y,z)\hat{\mathbf{x}} + A_y(x,y,z)\hat{\mathbf{y}} \\ &+ A_z(x,y,z)\hat{\mathbf{z}}\end{aligned}$$

Διαφορικά:

$d\mathbf{l} = dx\ \hat{\mathbf{x}} + dy\ \hat{\mathbf{y}} + dz\ \hat{\mathbf{z}}$

$d\mathbf{S} = dydz\ \hat{\mathbf{x}} + dxdz\ \hat{\mathbf{y}} + dxdy\ \hat{\mathbf{z}}$

$dV = dxdydz$

Παράγωγοι:

$$\begin{aligned}
\nabla &= \frac{\partial}{\partial x}\hat{\mathbf{x}} + \frac{\partial}{\partial y}\hat{\mathbf{y}} + \frac{\partial}{\partial z}\hat{\mathbf{z}} \\
\nabla f &= \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}} \\
\nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \\
\nabla^2\mathbf{A} &= \nabla^2 A_x\ \hat{\mathbf{x}} + \nabla^2 A_y\ \hat{\mathbf{y}} + \nabla^2 A_z\ \hat{\mathbf{z}} \\
\nabla\cdot\mathbf{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\
\nabla\times\mathbf{A} &= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} \\
&+ \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}
\end{aligned}$$

Κυλινδρικές: (ρ,ϕ,z)
Βαθμωτό πεδίο: $f(\rho,\phi,z)$
Διανυσματικό πεδίο:

$$\begin{aligned}\mathbf{A}(\rho,\phi,z) &= A_\rho(\rho,\phi,z)\,\hat{\boldsymbol{\varrho}} + A_\phi(\rho,\phi,z)\,\hat{\boldsymbol{\varphi}} \\ &+ A_z(\rho,\phi,z)\,\hat{\mathbf{z}}\end{aligned}$$

Διαφορικά:

$d\mathbf{l} = d\rho\ \hat{\boldsymbol{\varrho}} + \rho d\phi\ \hat{\boldsymbol{\varphi}} + dz\ \hat{\mathbf{z}}$

$d\mathbf{S} = \rho d\phi dz\ \hat{\boldsymbol{\varrho}} + d\rho dz\ \hat{\boldsymbol{\varphi}} + \rho d\rho d\phi\ \hat{\mathbf{z}}$

$dV = \rho d\rho d\phi dz$

Παράγωγοι:

$$\begin{aligned}
\nabla &= \frac{\partial}{\partial\rho}\hat{\boldsymbol{\varrho}} + \frac{1}{\rho}\frac{\partial}{\partial\phi}\hat{\boldsymbol{\varphi}} + \frac{\partial}{\partial z}\hat{\mathbf{z}} \\
\nabla f &= \frac{\partial f}{\partial\rho}\hat{\boldsymbol{\varrho}} + \frac{1}{\rho}\frac{\partial f}{\partial\phi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}} \\
\nabla^2 f &= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2} \\
\nabla^2\mathbf{A} &= \nabla^2 A_\rho\ \hat{\boldsymbol{\varrho}} + \nabla^2 A_\phi\ \hat{\boldsymbol{\varphi}} + \nabla^2 A_z\ \hat{\mathbf{z}} \\
\nabla\cdot\mathbf{A} &= \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z} \\
\nabla\times\mathbf{A} &= \left(\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\boldsymbol{\varrho}} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\hat{\boldsymbol{\varphi}} \\
&+ \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial\phi}\right]\hat{\mathbf{z}}
\end{aligned}$$

Σφαιρικές: (r,θ,ϕ)
Βαθμωτό πεδίο: $f(r,\theta,\phi)$
Διανυσματικό πεδίο:

$$\begin{aligned}\mathbf{A}(r,\theta,\phi) &= A_r(r,\theta,\phi)\,\hat{\mathbf{r}} + A_\theta(r,\theta,\phi)\,\hat{\boldsymbol{\vartheta}} \\ &+ A_\phi(r,\theta,\phi)\,\hat{\boldsymbol{\varphi}}\end{aligned}$$

Διαφορικά:

$d\mathbf{l} = dr\ \hat{\mathbf{r}} + rd\theta\ \hat{\boldsymbol{\vartheta}} + r\sin\theta d\phi\ \hat{\boldsymbol{\varphi}}$

$d\mathbf{S} = r^2\sin\theta d\theta d\phi\ \hat{\mathbf{r}} + r\sin\theta drd\phi\ \hat{\boldsymbol{\vartheta}} + rdrd\theta\ \hat{\boldsymbol{\varphi}}$

$dV = r^2\sin\theta drd\theta d\phi$

Παράγωγοι:

$$\begin{aligned}
\nabla &= \frac{\partial}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\boldsymbol{\vartheta}} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\hat{\boldsymbol{\varphi}} \\
\nabla f &= \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\boldsymbol{\vartheta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\hat{\boldsymbol{\varphi}} \\
\nabla^2 f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2} \\
\nabla^2\mathbf{A} &= \nabla^2 A_r\ \hat{\mathbf{r}} + \nabla^2 A_\theta\ \hat{\boldsymbol{\vartheta}} + \nabla^2 A_\phi\ \hat{\boldsymbol{\varphi}} \\
\nabla\cdot\mathbf{A} &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi} \\
\nabla\times\mathbf{A} &= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\hat{\boldsymbol{\vartheta}} \\
&+ \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\hat{\boldsymbol{\varphi}}
\end{aligned}$$
