Άσκηση 1

Αρμονικό Κύμα $y(x,t) = \underset{A}{0{,}4} \sin(\underset{k}{6}x - \underset{\omega}{2}t)$

α) Κατεύθυνση; / $u=;$

β) $A=;$, $\lambda=;$ $f=;$ $\varphi=;$

γ) Ν.δ.ο. ικανοποιεί την κυματική εξίσωση

δ) Συμβολή με όμοιο αλλά αντίθετης κατεύθυνσης κύμα
$y_{ολ}=;$ $\left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)\right]$ 3

α) Γενικά $y(x,t) = A\cdot\sin(kx \pm \omega t + \varphi)$

+ : αριστερά

− : δεξιά

Άρα εφόσον έχω − η κατεύθυνση είναι δεξιά

$u = \omega/k = \frac{2}{6}\, m/s \Rightarrow u = 1/3\, m/s$

β) $A = 0{,}4\, m$, $\lambda = \frac{2\pi}{k} = \frac{2\pi}{6}\, m = \frac{\pi}{3}\, m$

$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{6}{2\pi}\, Hz = \frac{3}{\pi}\, Hz$

$\varphi = 0$

γ) $\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2 y}{\partial t^2}}$ $\xrightarrow{(1),(2)}$ $-\cancel{0{,}4}\cdot 6\cdot 6\cdot \cancel{\sin(6x-2t)} = \frac{\cancel{-}1}{(1/3)^2}\cdot\cancel{0{,}4}\cdot(-2)(-2)\cancel{\sin(6x-2t)} \Rightarrow$

$\Rightarrow 3\cancel{6} = 9\cancel{4} \Rightarrow 1=1$ ✓

$\frac{\partial y}{\partial x} = \frac{\partial(0{,}4\cdot\sin(6x-2t))}{\partial x} = 0{,}4\cdot 6\cdot\cos(6x-2t)$ | $\frac{\partial y}{\partial t} = 0{,}4\cdot(-2)\cdot\cos(6x-2t)$

$\varphi = 0$

γ) $\boxed{\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{u^2}\dfrac{\partial^2 y}{\partial t^2}}$ $\xrightarrow{(1),(2)}$ $\cancel{-0},4\cdot 6\cdot 6\cdot \cancel{\sin(6x-2t)} = \dfrac{\cancel{-}1}{(1/3)^2}\cdot \cancel{0,4}\cdot(-2)(-2)\cancel{\sin(6x-2t)} \Rightarrow$

$\Rightarrow 3\cancel{6} = 9\cancel{4} \Rightarrow 1 = 1$ ✓

$\dfrac{\partial y}{\partial x} = \dfrac{\partial(0{,}4\cdot\sin(6x-2t))}{\partial x} = 0{,}4\cdot 6\cdot\cos(6x-2t)$

$\dfrac{\partial y}{\partial t} = 0{,}4\cdot(-2)\cdot\cos(6x-2t)$

$\dfrac{\partial^2 y}{\partial x^2} = -0{,}4\cdot 6\cdot 6\cdot\sin(6x-2t) \quad (1)$

$\dfrac{\partial^2 y}{\partial t^2} = -0{,}4\cdot(-2)\cdot(-2)\cdot\sin(6x-2t) \quad (2)$

δ) $\left.\begin{array}{l} y_1 = 0{,}4\cdot\sin(6x-2t) \\ y_2 = 0{,}4\cdot\sin(6x+2t)\end{array}\right\} \rightarrow y_1 + y_2 = 0{,}4\cdot\left[\sin\underbrace{(6x-2t)}_{A} + \sin\underbrace{(6x+2t)}_{B}\right] =$

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$= 0{,}4\left[2\sin\dfrac{6x-\cancel{2t}+6x+\cancel{2t}}{2}\cos\dfrac{\cancel{6x}-2t-\cancel{6x}-2t}{2}\right] = \underline{0{,}8\cdot\cos(2t)\cdot\sin(6x)}$

Στάσιμο Κύμα

$A\cos(\omega t)\cdot\sin(kx)$

Άσκηση 2

$$\overset{A}{\downarrow}\qquad\overset{k}{\downarrow}\quad\overset{\omega}{\downarrow}$$

Αρμονικό Κύμα $y(x,t) = 4\sin(3x-2t)$

α) Κατεύθυνση; / $\upsilon =$;

β) $A =$; , $\lambda =$; $f =$; $\varphi =$;

γ) Ν.δ.ο. ικανοποιεί την κυματική εξίσωση

δ) Συμβολή με όμοιο αλλά με διαφορά φάσης $\pi/4$

$y_{ολ} =$; $\left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)\right]$ (3)

Γενικά $y(x,t) = A\cdot\sin(kx \pm \omega t + \varphi)$

α) Εφόσον έχω $-$: κατεύθυνση δεξιά

$\upsilon = \omega/k = \frac{2}{3}$ m/s

β) $A = 4$ m, $\lambda = \frac{2\pi}{k} = \frac{2\pi}{3}$ m, $f = \frac{\omega}{2\pi} = \frac{1}{\pi}$ Hz

$\varphi = 0$

γ) $\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{\upsilon^2}\frac{\partial^2 y}{\partial t^2}}$ $\xrightarrow{(1),(2)}$ $\cancel{-4}\cdot 3\cdot\cancel{3}\cdot\cancel{\sin(3x-2t)} = \cancel{\frac{1}{(2/3)^2}}\,\cancel{4}\cdot\cancel{(-2)}(-2)\cdot\cancel{\sin(3x-2t)} \Rightarrow$

$\Rightarrow 1 = 1$ ✓

$\frac{\partial y}{\partial x} = \frac{\partial(4\sin(3x-2t))}{\partial x} = 4\cdot 3\cdot\cos(3x-2t)$ | $\frac{\partial y}{\partial t} = 4\cdot(-2)\cos(3x-2t)$

$y_{o\Sigma} = ;$ $\left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)\right]$ (3)

γ) $\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2 y}{\partial t^2}} \xrightarrow{(1),(2)} \cancel{-4}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{\sin(3x-2t)} = \frac{\cancel{1}}{\cancel{(2/3)^2}}\cancel{4}\cdot\cancel{(-2)}\cancel{(-2)}\cdot\cancel{\sin(3x-2t)} \Rightarrow$

$\Rightarrow 1 = 1$ ✓

$$\frac{\partial y}{\partial x} = \frac{\partial(4\sin(3x-2t))}{\partial x} = 4\cdot 3\cdot\cos(3x-2t)$$

$$\frac{\partial y}{\partial t} = 4\cdot(-2)\cos(3x-2t)$$

$$\frac{\partial^2 y}{\partial x^2} = -4\cdot 3\cdot 3\cdot\sin(3x-2t) \quad (1)$$

$$\frac{\partial^2 y}{\partial t^2} = -4(-2)\cdot(-2)\sin(3x-2t) \quad (2)$$

δ) $\left.\begin{aligned} y_1 &= 4\sin(3x-2t) \\ y_2 &= 4\sin\left(3x-2t+\frac{\pi}{4}\right)\end{aligned}\right\}$ $y_1+y_2 = 4\left[\sin(\underbrace{3x-2t}) + \sin(\underbrace{3x-2t+\frac{\pi}{4}})\right] \Rightarrow$

δ) $\left.\begin{aligned} y_1 &= 4\sin(3x-2t) \\ y_2 &= 4\sin\left(3x-2t+\frac{\pi}{4}\right) \end{aligned}\right\} \; y_1+y_2 = 4\left[\underbrace{\sin(3x-2t)}_{A} + \underbrace{\sin\left(3x-2t+\frac{\pi}{4}\right)}_{B}\right] \Rightarrow$

$$\overset{(3)}{\Rightarrow} y_1+y_2 = 4\left[2\cdot\sin\frac{3x-2t+3x-2t+\frac{\pi}{4}}{2}\cdot\cos\frac{\cancel{3x}-\cancel{2t}-\cancel{3x}+\cancel{2t}-\frac{\pi}{4}}{2}\right] =$$

$$= 8\sin\left(3x-2t+\frac{\pi}{8}\right)\cdot\cos\left(\frac{\pi}{8}\right) = \underline{8\cdot\cos\left(\frac{\pi}{8}\right)\sin\left(3x-2t+\frac{\pi}{8}\right)}$$

Γενικά $y_1+y_2 = 2A\cdot\cos(\varphi/2)\cdot\sin(kx-\omega t+\varphi/2)$