

Basic Identities:

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

$$(a+b)(a-b) = a^2 - b^2$$

Relation between roots and coefficients of a quadratic equation.

In general we can write a quadratic equation as follows:

$$ax^2 + bx + c = 0 \quad \text{or} \\ a \left[x^2 + \frac{b}{a}x + \frac{c}{a} \right] = 0 \quad \dots \dots \dots (1)$$

If r_1 and r_2 are roots of the above equation, then

$$a(x-r_1)(x-r_2) = 0 \quad \text{Expanding we get}$$

$$a \left[x^2 - r_1x - r_2x + r_1r_2 \right] = 0$$

$$a \left[x^2 - (r_1+r_2)x + r_1r_2 \right] = 0 \quad \dots \dots \dots (2)$$

Comparing (1) & (2) we get

$$\frac{b}{a} = -(r_1+r_2) \quad \text{or} \quad \boxed{r_1+r_2 = \frac{-b}{a}} \quad \leftarrow \text{SUM OF ROOTS}$$

$$\text{and} \quad \frac{c}{a} = r_1r_2 \quad \boxed{r_1r_2 = \frac{c}{a}} \quad \leftarrow \text{PRODUCT OF ROOTS}$$

Examples:

$$3x^2 - 2x - 4 = 0 \\ \text{SUM OF ROOTS} = \frac{-(-2)}{3} = \frac{2}{3} \\ \text{PRODUCT} = \frac{-4}{3} \\ \text{ROOTS ARE } \frac{1}{3} \pm \frac{\sqrt{13}}{3}$$

$$-2x^2 + x - 1 = 0 \\ \text{SUM OF ROOTS} = \frac{-1}{-2} = \frac{1}{2} \\ \text{PRODUCT} = \frac{-1}{-2} = \frac{1}{2} \\ \text{COMPLEX ROOTS}$$

$$x^2 - 3x + 2 = 0 \\ \text{SUM} = \frac{-(-3)}{1} = 3 \\ \text{PRODUCT} = 2 \\ \text{Roots are: } r_1 = 1 \\ r_2 = 2$$